Residual Inter-Contact Time for Opportunistic Networks with Pareto Inter-Contact Time: Two Nodes Case

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Abstract— *Opportunistic networks (OppNets) are appealing for many applications, such as wild life monitoring, disaster relief and mobile data offloading. In such a network, a message arriving at a mobile node could be transmitted to another mobile node when they opportunistically move into each other's transmission range (called in contact), and after multi-hop similar transmissions the message will finally reach its destination. Therefore, for one message the time interval from its arrival at a mobile node to the time the mobile node contacts another node constitutes an essential part of the message's whole delay. Thus, studying stochastic properties of this time interval between two nodes lays a solid foundation for evaluating the whole message delay in OppNets. Note that this time interval is within the time interval between two consecutive node contacts (called inter-contact time) and it is also referred to as residual inter-contact time. In this paper, we derive the closed-form distribution for residual inter-contact time. First, we formulate the contact process of a pair of mobile nodes as a renewal process, where the inter-contact time features the popular Pareto distribution. Then, we derive, based on the renewal theory, closed-form results for the transient distribution of residual inter-contact time and also its limiting distribution. Our theoretical results on distribution of residual inter-contact time are validated by simulations.*

Keywords: Opportunistic networks, DTNs, inter-contact times

1. Introduction

Nowadays, portable mobile nodes (e.g., smart phones, tablets, digital cameras, censors) have been used ubiquitously in our daily life. Equipped with advanced wireless communication technologies (e.g., Bluetooth, WiFi Direct and ZigBee), these mobile nodes are now able to communicate directly with each other when they opportunistically move into transmission range (also called in contact). This promises a novel communication paradigm, opportunistic networks (OppNets)[1], which exploit opportunistic direct contacts of mobile nodes to deliver messages among them [1], shown in Fig.1a. Since OppNets are cost-effective, resilient to node failures and can be deployed rapidly, they can be used to enable communications in extreme environments (e.g., disaster, rural areas and wildlife monitoring) and enhance communications in existing networks (e.g., offloading data traffic in cellular networks, where mobile nodes share data directly when in contact).

[1]OppNets are also referred to as delay tolerant networks (DTNs).

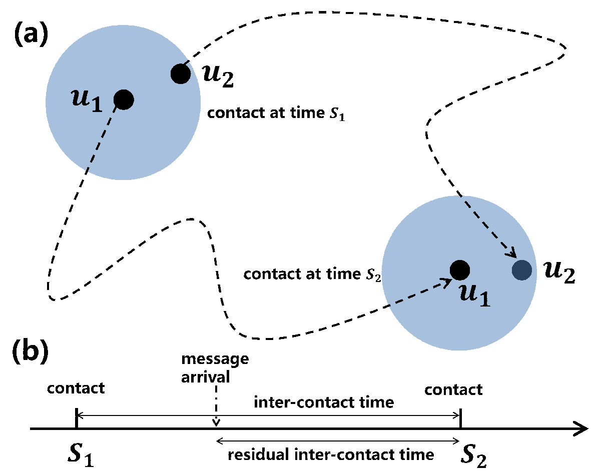

Fig. 1: (a) Direct communication when node u_1 contacts u_2. (b) Relationship between residual inter-contact time and inter-contact time.

In OppNets, a message arriving at a mobile node is transmitted directly to another mobile node when the two nodes opportunistically contact each other, and after multi-hop similar transmissions the message will finally reach its destination. Therefore, for one message the time interval from its arrival at a mobile node to the time the mobile node contacts another node constitutes an essential part of the whole delay of that message and thus significantly impacts the message's delay performance. Note that this time interval is within the time interval between two consecutive node contacts (called inter-contact time) as shown in Fig.1b, and it is also referred to as residual inter-contact time. Since residual inter-contact time represents the time a message has to wait at a mobile node before getting transmitted to next mobile node, its study serves as a cornerstone for evaluating the whole message delay.

As residual inter-contact time is embedded in inter-contact time, extensive works have been done first on investigating inter-contact time distribution. By analyzing real mobility traces of OppNets [2]–[4], the authors [5], [6] found that Pareto distribution with or without exponential cutoff is a good fitting distribution for inter-contact time there. By analyzing synthetic mobility models (such as random waypoint, random walk), the authors [6], [7] also concluded that Pareto distribution is a reasonable distribution for characterizing inter-contact time (refer to Section 2 for more related works on inter-contact time). Based on such findings, researchers studied the distribution of residual inter-contact time for OppNets with Pareto inter-contact time. The authors [5] derived upper and lower bounds for the distribution of residual inter-contact time. Later, the authors [6] presented a general expression for calculating the accurate distribution of residual inter-contact time, but arrived at a wrong distribution result for OppNets with Pareto inter-contact time [8]. The authors [8] attempted to derive the correct distribution of residual inter-contact time, however, they used the limiting time-average fraction of residual time less than given value to represent the real distribution of residual inter-contact time, which is not justified.

Our contribution in this paper is to rigorously derive the accurate distribution of residual inter-contact time for homogeneous OppNets, where inter-contact times of all node pairs follow the same Pareto distribution. Our work also justifies the result in [8].

- First, we formulate the contact process of a pair of mobile nodes as a renewal process, where the inter-contact time features the popular Pareto distribution [5], [9].
- Then, we derive, based on the renewal theory, closed-form results for transient distribution of residual inter-contact time and also its limiting distribution as time goes to infinity. For a tagged node, we also derive the distribution of the shortest residual inter-contact time between that node and all other nodes.
- Finally, we conduct simulations to validate the theoretical results on distribution of residual inter-contact time.

Our results on distribution of residual inter-contact time can be used to analyze delay performance of popular routing protocols in OppNets, such as epidemic routing and two-hop relay routing protocols.

The rest of this paper is organized as follows. In Section 2 we give more related works on inter-contact times. In Section 3, we present problem formulation by rigorously defining inter-contact time, residual inter-contact time and relative quantities. We then derive both transient and limiting distributions for residual inter-contact time in Section 4. We present simulation results to validate our derived distribution in Section 5. Finally, we conclude this paper in Section 6.

2. Related Works

Cai and Eun [10] investigated the age of inter-contact time between two mobile nodes, which could be used to study their residual inter-contact time properties. Passarella and Conti [11] characterized the relationship between the distributions of inter-contact times of different node pairs and the resulting aggregate distribution (the distribution obtained from aggregating samples of inter-contact times of all node pairs) in heterogeneous opportunistic networks where inter-contact times of different node pairs follow different distributions. Through empirical statistical analysis, Zhu and others [12] reported that inter-contact times of vehicles follow exponential distribution. By modeling general synthetic mobility model, La [13] showed that the distribution of inter-contact times can be well approximated by an exponential distribution under some conditions. Most works in the literature on inter-contact times used aggregated samples of inter-contact times of all nodes to estimate inter-contact time distribution of a pair of nodes, however, Orallo and others [14] showed this method cannot accurately characterize pair-wise inter-contact time distribution. Instead, they proposed another two methods (namely, aggregate nodes and any contact) to better characterize inter-contact time distribution. Biondi and others [15] studied the effect of power saving policy (duty cycling) on the performances of inter-contact times between mobile devices and showed that the inter-contact times under duty cycling are approximately exponential when original inter-contact times of devices are exponential.

3. Problem Formulation

Suppose two mobile nodes u_1 and u_2 move around in an OppNet, they employ the same wireless transmission range as shown in Fig.1a. We define the following terms for the two nodes.

Contact: We say node u_1 contacts node u_2 if u_2 moves into the wireless transmission rang of u_1, as illustrated in Fig.1a.

Contact Epoch: A contact epoch is the time instant at which u_1 contacts u_2. We denote successive contact epochs of u_1 and u_2 by $S_0, S_1, S_2, \cdots$ where $0 = S_0 < S_1 < S_2 < \cdots$.

Inter-Contact Time: An inter-contact time X for u_1 and u_2 is the time interval between their two consecutive contacts, as shown in Fig.1b. Thus, the i-th inter-contact time $X_i = S_i - S_{i-1}$, where $i \geq 1$. We assume X_i's are independent and identically distributed (IID) random variables as previous works [5]–[7].

Contact Process: The contact process of u_1 and u_2 is denoted by $\{N(t); t > 0\}$ where $N(t)$ records the number of total contacts between u_1 and u_2 occurring in time interval $(0, t]$, i.e., up to and including time t. Then, $N(t) = n$ if and only if $S_n \leq t < S_{n+1}$, as shown in Fig.2. Note also that if u_1 and u_2 contact at time t then $S_{N(t)} = t$. Since all inter-contact times are IID, the contact process is actually a renewal process (a renewal process is an arrival process where all inter-arrival intervals are positive IID random variables) [16].

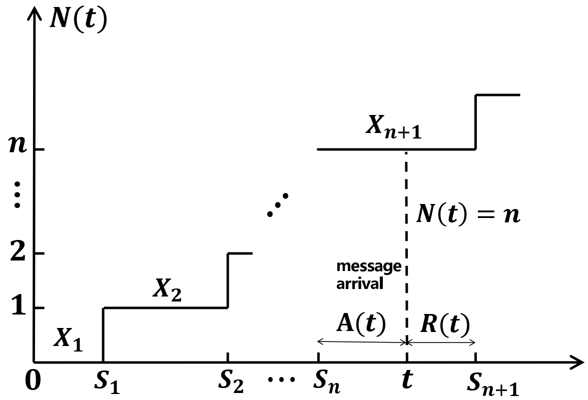

Fig. 2: Sample path of contact process $N(t)$ for u_1 and u_2.

Residual Inter-Contact Time: Assume a message arrives at u_1 at time t, then the time interval from t to the next time node u_1 contacts node u_2 is defined to be the residual inter-contact time for that message at time t, which is denoted by $R(t)$ and $R(t) = S_{N(t)+1} - t > 0$, i.e., the time interval within the inter-contact time as shown in Fig.2.

Age of Inter-Contact Time: Assume a message arrives at u_1 at time t, then the time interval from the most recent contact between u_1 and u_2 before/at t to time t is defined to be the the age of inter-contact time for that message at time t, which is denoted by $A(t)$ and $A(t) = t - S_{N(t)} \geq 0$, as shown in Fig.2.

Note that if $N(t) = 0$ at time t, it means that no contact has happened between u_1 and u_2 in time interval $(0, t]$, then the first inter-contact time X_1 must satisfy $X_1 > t$ and consequently $A(t) = t - S_0 = t$; if $N(t) \geq 1$ at time t, then $S_{N(t)} > 0$ and $A(t) = t - S_{N(t)} < t$. To sum up, $0 \leq A(t) \leq t$.

Concerned Inter-Contact Time: Assume a message arrives at u_1 at time t, then the time interval from the most recent contact epoch of u_1 and u_2 before/at t, $S_{N(t)}$, to the next time they contact each other, $S_{N(t)+1}$, is defined to be the concerned inter-contact time, which is denoted by $\widetilde{X}(t)$ and $\widetilde{X}(t) = S_{N(t)+1} - S_{N(t)} = X_{N(t)+1} > 0$.

From the definitions of $R(t)$, $A(t)$ and $\widetilde{X}(t)$, we have the followings: $\widetilde{X}(t) = R(t) + A(t)$ and $\widetilde{X}(t) > A(t)$, for any $t \geq 0$.

Pareto Inter-Contact Time: Analysis of real mobility traces and synthetic mobility models suggests that Pareto distribution can well approximate the distribution of inter-contact times in OppNets [5]–[7]. Thus, we assume all inter-contact times X_i's for node u_1 and u_2 feature a Pareto distribution with scalar parameters $x_m > 0$ and $\alpha > 1$ as follow [6], [8],

$$F_X(x) = Pr\{X \leq x\} = \begin{cases} 1 - \left(\frac{x_m}{x}\right)^{\alpha} & \text{if } x \geq x_m, \\ 0 & \text{if } 0 < x < x_m, \end{cases} \quad (1)$$

from which we also have the mean value

$$\overline{X} = \mathbb{E}\{X\} = \frac{\alpha x_m}{\alpha - 1}. \quad (2)$$

4. Inter-Contact Time Analysis

In this section we derive closed-form results for the distribution of residual inter-contact time. We first present a lemma below, which will be used in our following derivation.

Lemma 1: Consider the contact process $\{N(t); t > 0\}$ between u_1 and u_2 defined before. Suppose a message arrives at u_1 at time t. For given constant a, δ and x satisfying $0 \leq a < a + \delta \leq t$, $a + 2\delta \leq x$, let E denote the following event

$$E = \{a \leq A(t) < a + \delta, x - \delta < \widetilde{X}(t) \leq x\}, \quad (3)$$

where $A(t)$ is the age of inter-contact time at time t for that message, $\widetilde{X}(t)$ is the concerned inter-contact time.

Then, we have

$$Pr\{E\} = \big(m(t-a) - m(t-a-\delta)\big)\big(F_X(x) - F_X(x-\delta)\big) \quad (4)$$

where $m(t) = \mathbb{E}\{N(t)\}$.

Proof: Since the contact process forms a renewal process, this lemma follows directly from Theorem 5.7.2 [16]. ■

Now, we derive the transient distribution of residual inter-contact time for nodes u_1 and u_2.

Theorem 1: For an OppNet with Pareto inter-contact times given in (1), the transient distribution of residual inter-contact time $R(t)$ for a message at time t is

$$Pr\{R(t) \leq r\} = \Big(\frac{x_m}{t}\Big)^{\alpha} - \Big(\frac{x_m}{t+r}\Big)^{\alpha} + \int_0^t \big(F_X(t-\tau+r) - F_X(t-\tau)\big)\mathrm{d}m(\tau) \quad (5)$$

where $r > 0$.

Proof: Assume a message arrives at node u_1 at time t. Then,

$$\begin{aligned} & Pr\{R(t) \leq r\} && (6)\\ =& Pr\{\widetilde{X}(t) - A(t) \leq r\} && (7)\\ =& \underbrace{Pr\{\widetilde{X}(t) - A(t) \leq r, A(t) = t\}}_{P_1} \\ &+ \underbrace{Pr\{\widetilde{X}(t) - A(t) \leq r, 0 \leq A(t) < t\}}_{P_2} && (8)\end{aligned}$$

where (8) follows from the fact that $0 \leq A(t) \leq t$ and the law of total probability.

We calculate probability P_1 first. Recall that $\widetilde{X}(t)$ is the concerned inter-contact time containing time t and $\widetilde{X}(t) >$

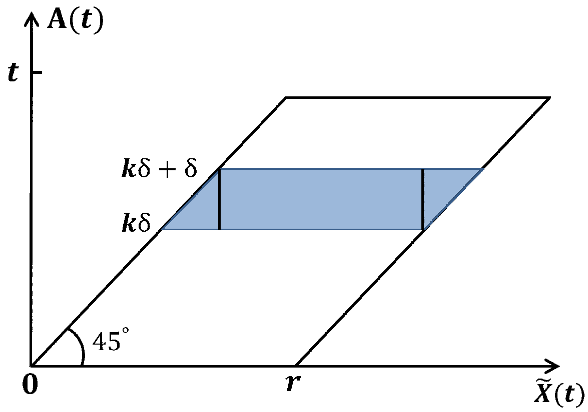

Fig. 3: Sample points of joint $A(t)$ and $\widetilde{X}(t)$.

$A(t)$ for any t.

$$P_1 = Pr\{A(t) < \widetilde{X}(t) \le A(t)+r, A(t)=t\} \quad (9)$$
$$= Pr\{t < \widetilde{X}(t) \le t+r, A(t)=t\} \quad (10)$$
$$= Pr\{t < X_{N(t)+1} \le t+r, A(t)=t\} \quad (11)$$
$$= Pr\{t < X \le t+r\} \quad (12)$$
$$= F_X(t+r) - F_X(t) \quad (13)$$
$$= \left(\frac{x_m}{t}\right)^{\alpha} - \left(\frac{x_m}{t+r}\right)^{\alpha} \quad (14)$$

where (12) follows from the fact that $A(t) = t$ indicates no contact happens in $(0, t]$, i.e., $N(t) = 0$, thus, $\widetilde{X}(t) = X_{N(t)+1} = X_1$. Note also that all X_i's follow the same Pareto distribution given in (1).

We next calculate probability P_2.

$$P_2 = Pr\{A(t) < \widetilde{X}(t) \le A(t)+r, 0 \le A(t) < t\} \quad (15)$$
$$= \sum_{k=0}^{l-1} \underbrace{Pr\{A(t) < \widetilde{X}(t) \le A(t)+r, k\delta \le A(t) < k\delta+\delta\}}_{\widetilde{P}_k} \quad (16)$$

where we divide interval $0 \le A(t) < t$ into l sub-intervals $[k\delta, k\delta+\delta)$, $0 \le k \le l-1$, and $\delta = \frac{t}{l}$.

Next, we calculate the general term $\widetilde{P}_k$ in (16),

$$\widetilde{P}_k = Pr\{A(t) < \widetilde{X}(t) \le A(t)+r, k\delta \le A(t) < k\delta+\delta\} \quad (17)$$

Note that $\widetilde{P}_k$ is the probability of the event illustrated by the gray area of sample points of age $A(t)$ and $\widetilde{X}(t)$, shown in Fig.3. From this figure, we see that

$$\widetilde{P}_k \ge Pr\{k\delta+\delta < \widetilde{X}(t) \le k\delta+r, k\delta \le A(t) < k\delta+\delta\}, \quad (18)$$
$$= \big(m(t-k\delta) - m(t-k\delta-\delta)\big) \cdot \big(F_X(k\delta+r) - F_X(k\delta+\delta)\big), \quad (19)$$

where (19) follows from Lemma 1. Similarly,

$$\widetilde{P}_k \le Pr\{k\delta < \widetilde{X}(t) \le k\delta+r+\delta, k\delta \le A(t) < k\delta+\delta\}, \quad (20)$$
$$= \big(m(t-k\delta) - m(t-k\delta-\delta)\big) \cdot \big(F_X(k\delta+r+\delta) - F_X(k\delta)\big). \quad (21)$$

Thus, from (15), (19) and (21), we have

$$P_2 \ge \underbrace{\sum_{k=0}^{l-1} \big(m(t-k\delta) - m(t-k\delta-\delta)\big) \cdot F_X(k\delta+r)}_{L_1} - \underbrace{\sum_{k=0}^{l-1} \big(m(t-k\delta) - m(t-k\delta-\delta)\big) \cdot F_X(k\delta+\delta)}_{U_2}, \quad (22)$$

and

$$P_2 \le \underbrace{\sum_{k=0}^{l-1} \big(m(t-k\delta) - m(t-k\delta-\delta)\big) \cdot F_X(k\delta+r+\delta)}_{U_1} \quad (23)$$
$$- \underbrace{\sum_{k=0}^{l-1} \big(m(t-k\delta) - m(t-k\delta-\delta)\big) \cdot F_X(k\delta)}_{L_2}, \quad (24)$$

where U_1 and L_1 are just the upper and lower Stieltjes sums of $F_X(t-\tau+r)$ with respect to $m(\tau)$ on $[0, t]$, U_2 and L_2 are the upper and lower Stieltjes sums of $F_X(t-\tau)$ with respect to $m(\tau)$ on $[0, t]$.

Since $m(\tau)$ is the expectation of $N(t)$, $m(\tau)$ is an increasing function on $[0, t]$ and thus is of bounded variation on $[0, t]$ (Theorem 6.5 [17]). Note also that $F_X(t-\tau+r)$ is a continuous function on $[0, t]$ since $F_X(x)$ is Pareto distribution. These two conditions indicate the existence of the following Riemann-Stieltjes integral (Theorem 7.27 [17])

$$\int_0^t F_X(t-\tau+r)\mathrm{d}m(\tau). \quad (25)$$

The existence of Riemann-Stieltjes integral in (25) further indicates that (Theorem 7.19 [17]) for any $\epsilon_1 > 0$,

$$0 \le U_1 - L_1 < \epsilon_1, \quad \text{as} \quad l \to \infty. \quad (26)$$

Similarly, the following Riemann-Stieltjes integral also exists

$$\int_0^t F_X(t-\tau)\mathrm{d}m(\tau) \quad (27)$$

and for any $\epsilon_2 > 0$,

$$0 \le U_2 - L_2 < \epsilon_2, \quad \text{as} \quad l \to \infty. \quad (28)$$

From (26) and (28), we know that for any $\epsilon > 0$,

$$0 \leq (U_1 - L_2) - (L_1 - U_2) < \epsilon, \quad \text{as} \quad l \to \infty. \tag{29}$$

Thus, according to Theorem 7.19 [17], we have

$$P_2 = \int_0^t \big(F_X(t - \tau + r) - F_X(t - \tau)\big)\mathrm{d}m(\tau) \tag{30}$$

■

Next, we derive the limiting distribution of residual inter-contact time $R(t)$ as $t \to \infty$.

Theorem 2: For an opportunistic network with Pareto inter-contact time given in (1), the limiting distribution of residual inter-contact time $R(t)$ of a message as $t \to \infty$ is

$$\lim_{t\to\infty} Pr\{R(t) \leq r\} = \begin{cases} 1 - \frac{1}{\alpha}\left(\frac{x_m}{r}\right)^{\alpha-1} & \text{if } r \geq x_m, \\ \frac{r\alpha - r}{\alpha x_m} & \text{if } 0 < r < x_m. \end{cases} \tag{31}$$

Proof: From (5), we know

$$\begin{aligned} &\lim_{t\to\infty} Pr\{R(t) \leq r\} \\ &= \lim_{t\to\infty} \int_0^t \big(F_X(t - \tau + r) - F_X(t - \tau)\big)\mathrm{d}m(\tau). \end{aligned} \tag{32}$$

From the key renewal theorem [16], we know

$$\begin{aligned} &\lim_{t\to\infty} \int_0^t \big(F_X(t - \tau + r) - F_X(t - \tau)\big)\mathrm{d}m(\tau) \\ &= \frac{1}{\overline{X}} \int_0^\infty \big(F_X(x + r) - F_X(x)\big)\mathrm{d}x \end{aligned} \tag{33}$$

where $F_X(x)$ and $\overline{X}$ are given in (1) and (2), respectively.

We next calculate the integral in (33). For $0 < r \leq x_m$,

$$F_X(x + r) = \begin{cases} 1 - \left(\frac{x_m}{x+r}\right)^\alpha & \text{if } x + r \geq x_m, \\ 0 & \text{if } 0 < x + r < x_m. \end{cases} \tag{34}$$

Thus,

$$\begin{aligned} &F_X(x + r) - F_X(x) \\ &= \begin{cases} \left(\frac{x_m}{x}\right)^\alpha - \left(\frac{x_m}{x+r}\right)^\alpha & \text{if } x > x_m, \\ 1 - \left(\frac{x_m}{x+r}\right)^\alpha & \text{if } x_m - r < x \leq x_m, \\ 0 & \text{if } 0 < x \leq x_m - r. \end{cases} \end{aligned} \tag{35}$$

Then, we have

$$\begin{aligned} &\int_0^\infty \big(F_X(x + r) - F_X(x)\big)\mathrm{d}x \\ &= \int_{x_m - r}^{x_m} \Big(1 - \Big(\frac{x_m}{x+r}\Big)^\alpha\Big)\mathrm{d}x + \int_{x_m}^\infty \Big(\Big(\frac{x_m}{x}\Big)^\alpha - \Big(\frac{x_m}{x+r}\Big)^\alpha\Big)\mathrm{d}x \end{aligned} \tag{36}$$

$$= \quad r \tag{37}$$

For $r > x_m$, since $x + r > x_m$ for any $x > 0$, we have

$$F_X(x + r) = 1 - \left(\frac{x_m}{x + r}\right)^\alpha, \; x > 0. \tag{38}$$

Thus,

$$\begin{aligned} &F_X(x + r) - F_X(x) \\ &= \begin{cases} \left(\frac{x_m}{x}\right)^\alpha - \left(\frac{x_m}{x+r}\right)^\alpha & \text{if } x > x_m, \\ 1 - \left(\frac{x_m}{x+r}\right)^\alpha & \text{if } 0 < x \leq x_m. \end{cases} \end{aligned} \tag{39}$$

Then, we have

$$\begin{aligned} &\int_0^\infty \big(F_X(x + r) - F_X(x)\big)\mathrm{d}x \\ &= \int_0^{x_m} \Big(1 - \Big(\frac{x_m}{x+r}\Big)^\alpha\Big)\mathrm{d}x + \int_{x_m}^\infty \Big(\Big(\frac{x_m}{x}\Big)^\alpha - \Big(\frac{x_m}{x+r}\Big)^\alpha\Big)\mathrm{d}x \end{aligned} \tag{40}$$

$$= x_m + \frac{x_m^\alpha r^{-\alpha+1}}{-\alpha + 1} - \frac{x_m}{-\alpha + 1} \tag{41}$$

To sum up,

$$\int_0^\infty \big(F_X(x + r) - F_X(x)\big)\mathrm{d}x \tag{42}$$

$$= \begin{cases} x_m + \frac{x_m^\alpha r^{-\alpha+1}}{-\alpha+1} - \frac{x_m}{-\alpha+1} & \text{if } r > x_m, \\ r & \text{if } 0 < r \leq x_m. \end{cases} \tag{43}$$

After substituting (2) and (43) into (33), we have

$$\lim_{t\to\infty} Pr\{R(t) \leq r\} = \begin{cases} 1 - \frac{1}{\alpha}\left(\frac{x_m}{r}\right)^{\alpha-1} & \text{if } r > x_m, \\ \frac{r\alpha - r}{\alpha x_m} & \text{if } 0 < r \leq x_m. \end{cases} \tag{44}$$

This completes the proof. ■

Finally, we want to find out how long it will take a node with a new arrival message to contact another node in the OppNet, thus having opportunities to forward the message to the next node.

Suppose a homogeneous opportunistic network of W nodes, where all nodes move independently and the inter-contact times of every pair of nodes feature the same Pareto distribution given in (1). Let $R_{ij}(t)$ be the residual inter-contact time for node i and node j, then the time interval $R^*(t)$ from time t the message arrives at node u_1 to the time u_1 contacts any one of the other $W - 1$ nodes in the network is

$$R^*(t) = \min\big\{R_{12}(t), R_{13}(t), \cdots, R_{1W}(t)\big\} \tag{45}$$

Lemma 2: The limiting distribution of $R^*(t)$ is given as follows.

$$\begin{aligned} &\lim_{t\to\infty} Pr\{R^*(t) \leq r\} \\ &= \begin{cases} 1 - \left(\frac{1}{\alpha}\right)^{W-1}\left(\frac{x_m}{r}\right)^{(\alpha-1)(W-1)} & \text{if } r \geq x_m, \\ 1 - \left(\frac{\alpha x_m - r\alpha + r}{\alpha x_m}\right)^{W-1} & \text{if } 0 < r < x_m. \end{cases} \end{aligned} \tag{46}$$

Proof:

$$Pr\{R^*(t) \leq r\} = 1 - Pr\{R^*(t) > r\} \tag{47}$$

$$= 1 - Pr\{R_{12}(t) > r, R_{13}(t) > r, \cdots, R_{1W}(t) > r\} \tag{48}$$

$$= 1 - Pr\{R_{12}(t) > r\}Pr\{R_{13}(t) > r\} \cdots Pr\{R_{1W}(t) > r\} \tag{49}$$

where (49) follows from the independence of node mobility. After substituting (31) into (49), we got (46). ■

5. Simulation Results

To validate the derived distribution of residual inter-contact time, we developed a customized simulator in C++ to simulate the contact process between two nodes u_1, u_2, the random message arrival process to node u_1, and observe the residual inter-contact times regarding message arrivals. Specifically, we simulated three different network scenarios where inter-contact times between u_1 and u_2 all follow Pareto distribution but with different scalar parameter settings: $(x_m = 1.0, \alpha = 1.5)$, $(x_m = 1.0, \alpha = 2.0)$ and $(x_m = 2.0, \alpha = 3.0)$. We assume messages arrive at node u_1 according to a Poisson process with arrival rate of 0.001. During simulations, we measured the residual inter-contact time for a message as the time interval from the time it arrives at u_1 to the next time u_1 contacts u_2. From the measured residual inter-contact times, we calculated their distribution. The simulated distributions under three network scenarios are presented in Fig.4, where corresponding theoretical distributions are also given for comparison. From Fig.4, we can see that our derived distributions for residual inter-contact time perfectly match the simulated ones, verifying our theoretical results.

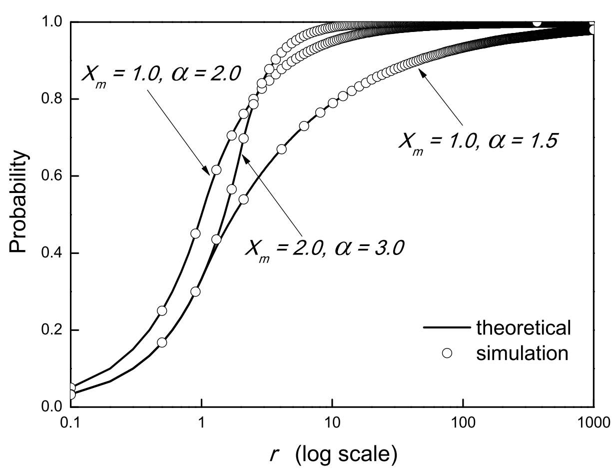

Fig. 4: Disbribution of residual inter-contact time under different Pareto distribution parameters.

6. Conclusion

In this paper, we rigorously derived the distribution of residual inter-contact time for opportunistic networks with Pareto inter-contact times. Our results have important implications for applications (by the law of large numbers): for a homogeneous OppNet, where contacts of all node pairs follow common inter-contact time distribution (e.g., students in a campus and corporate users [18]), the distribution of residual inter-contact times can be found out by collecting samples of residual inter-contact times from all node pairs in stead of collecting samples from the same node pair for a long time. This creates great experiment convenience, since long-time tracking of the same pair of nodes is usually prohibited due to privacy while short-time tracking all node pairs is much easier.

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