SURFACE COMPLETION BY MINIMIZING ENERGY BASED ON SIMILARITY OF SHAPE

Norihiko Kawai, Tomokazu Sato and Naokazu Yokoya

Graduate School of Information Science, Nara Institute of Science and Technology
8916-5 Takayama, Ikoma, Nara 630-0192, Japan
{norihi-k,tomoka-s,yokoya}@is.naist.jp

ABSTRACT

3D mesh models generated with range scanner or video images often have holes due to many occlusions by other objects and the object itself. This paper proposes a novel method to fill the missing parts in the incomplete models. The missing parts are filled by minimizing the energy function, which is defined based on similarity of local shape between the missing region and the rest of the object. The proposed method can generate complex and consistent shapes in the missing region. In the experiment, the effectiveness of the method is successfully demonstrated by applying it to complex shape objects with missing parts.

***Index Terms*—** Surface Completion, 3D Inpainting, Energy Minimization, Pattern Similarity

1. INTRODUCTION

3D models of real environments are widely used for entertainment and digital archives. Thus, automatic generation of 3D models by using range scanner and video images has been investigated. Such methods can obtain whole 3D models by measuring the target objects from multiple view points and integrate the partial shapes. However, when the target is a large and complex environment such as an outdoor scene, it is difficult to generate the complete 3D models without holes due to many occlusions. For this problem, lots of surface completion methods which fill the missing regions and obtain complete 3D mesh models have been proposed. These methods are classified into two categories: One is based on considering the smoothness of surface shape and the other is based on using example shapes.

As methods considering the smoothness of surface shape, partial differential equation, moving least squares and Willmore flow have been used for filling holes [1, 2, 3]. These methods can fill the missing regions with smooth surface patches. Although they are effective for small holes in a 3D model, unnatural shapes may be generated when the missing parts are large and surrounding shape is complex because the methods cannot generate a complex surface. As another approach considering the smoothness, volumetric-based methods have been proposed [4, 5]. Davis et al. [4] have filled holes by diffusing the signed distance function. This method largely depends on the location and the number of measuring points. Thus, it is difficult to obtain good results when the location is grossly one-sided and the number is small. Podolak et al. [5] have used the graph cut to interpolate a surface. Though this method can generate a continuous surface, to generate complex shapes in the missing region, user's manual inputs are required.

On the other hand, in order to generate complex shapes in missing regions, the methods using example shapes on other parts of the object (data region) have been proposed [6, 7, 8, 9, 10]. These methods calculate the similarity between the local surface shape around the missing regions and that in the data region and fill the missing regions by copying the most similar surface patches successively. The methods can generate complex surfaces. However, a discontinuous surface is easily generated on the seam in the completed model.

In this paper, to solve these problems, the energy function which represents the non-plausibility of surface shape is defined using the similarity between the local surface shapes of the missing and data regions. By minimizing the energy for whole the missing region, the model without discontinuous surface shapes can be generated as an optimal solution.

2. SURFACE COMPLETION BY MINIMIZING AN ENERGY FUNCTION

Fig. 1 illustrates the flow diagram of the proposed method. First, a user manually selects missing regions to be repaired in a model (a). Next, initial points and faces are generated to the missing regions (b). Finally, by repeating addition and deletion of points considering density of points (c) and updating positions of points by minimizing energy (d), whole surface is optimally completed. In the following sections, first, the energy function is defined based on local shape similarity SSD in Section 2.1. Next, Section 2.2 describes the procedure that minimizes the energy function.

2.1. Energy function based on similarity of local shape

As illustrated in Fig. 2, first, a 3D model is divided into region Ω' including the missing region Ω selected by a user and the data region Φ which is the rest of the object. The plausibility in the missing region Ω is defined by using surface shape in

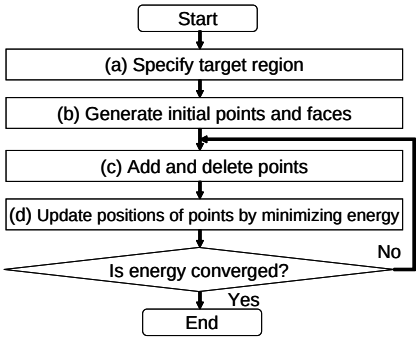


Fig. 1. Flow diagram of the proposed method.

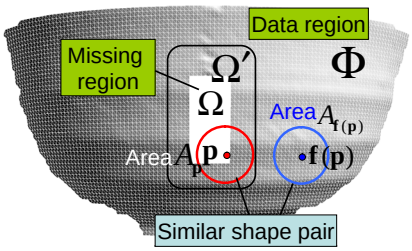


Fig. 2. Missing and data regions in a 3D model.

the data region Φ. Here, Ω' is the expanded area of region Ω. Ω' is determined so that a spherical area A with constant radius in the region Ω' includes at least one of the points in the region Ω. The energy function is defined as the weighed sum of SSD between the point set around the point $\mathbf{p}$ in the region Ω' and surface shape around the point $\mathbf{f}(\mathbf{p})$ in the region Φ as follows:

$$E = \frac{\sum_{\mathbf{p}\in\Omega'} w_{\mathbf{p}} SSD(\mathbf{p}, \mathbf{f}(\mathbf{p}))}{\sum_{\mathbf{p}\in\Omega'} w_{\mathbf{p}}}. \quad (1)$$

Here, the weight $w_{\mathbf{p}}$ is set as 1 if $\mathbf{p}$ is inside of the region $\Omega' \cap \overline{\Omega}$ because positions of points in this region are fixed: otherwise $w_{\mathbf{p}}$ is set as $c_1^{-c_2 d}$ (d is the distance from the boundary of Ω, and c_1 and c_2 are constants) because positions of points around the boundary have higher confidence than those in the center of the missing region. In Eq. (1), the weight is normalized because the weight of each point change due to the movement of points. $\mathbf{f}(\mathbf{p})$ in the data region Φ denotes the central point of the spherical area whose local shape is the most similar to that of point $\mathbf{p}$. The point $\mathbf{f}(\mathbf{p})$ for minimizing E is determined as follows:

$$\mathbf{f}(\mathbf{p}) = \underset{\mathbf{p}'\in\Phi}{\operatorname{argmin}}(SSD(\mathbf{p}, \mathbf{p}')). \quad (2)$$

In this paper, $SSD(\mathbf{p}, \mathbf{f}(\mathbf{p}))$ that represents the similarity of local shape between the missing and data regions is defined as the sum of distances between the points in the area $A_{\mathbf{p}}$ whose central point is $\mathbf{p}$ and the aligned surface around $\mathbf{f}(\mathbf{p})$ as follows:

$$SSD(\mathbf{p}, \mathbf{f}(\mathbf{p})) = \frac{1}{N(A_{\mathbf{p}})} \sum_{\mathbf{p}_i \in A_{\mathbf{p}}} \| \mathbf{p}_i - \mathbf{M}_{\mathbf{f}(\mathbf{p})\mathbf{p}} \mathbf{g}(\mathbf{p}_i) \|^2, \quad (3)$$

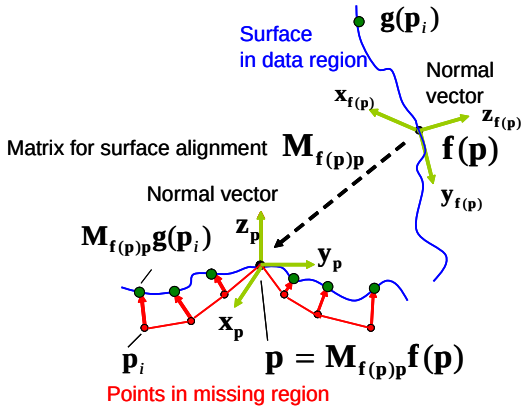


Fig. 3. Alignment of point clouds and surface for similarity measure of local shape.

where $\mathbf{M}_{\mathbf{f}(\mathbf{p})\mathbf{p}}$ denotes the matrix for surface alignment as shown in Fig. 3. $\mathbf{M}_{\mathbf{f}(\mathbf{p})\mathbf{p}}\mathbf{g}(\mathbf{p}_i)$ is a point on the aligned surface in the data region which exists in the normal direction of the point $\mathbf{p}_i$ ($\in A_{\mathbf{p}}$). $N(A_{\mathbf{p}})$ is the number of points in the spherical area $A_{\mathbf{p}}$.

Matrix for surface alignment $\mathbf{M}_{\mathbf{f}(\mathbf{p})\mathbf{p}}$ is defined as follows:

$$\mathbf{M}_{\mathbf{f}(\mathbf{p})\mathbf{p}} = \begin{pmatrix} \mathbf{x}_{\mathbf{p}} & & -x_{\mathbf{p}} \\ \mathbf{y}_{\mathbf{p}} & & -y_{\mathbf{p}} \\ \mathbf{z}_{\mathbf{p}} & & -z_{\mathbf{p}} \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{x}_{f(\mathbf{p})} & & -x_{\mathbf{f}(\mathbf{p})} \\ \mathbf{y}_{f(\mathbf{p})} & & -y_{\mathbf{f}(\mathbf{p})} \\ \mathbf{z}_{f(\mathbf{p})} & & -z_{\mathbf{f}(\mathbf{p})} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (4)$$

where $(\mathbf{x}_{\mathbf{p}}, \mathbf{y}_{\mathbf{p}}, \mathbf{z}_{\mathbf{p}})$ and $(\mathbf{x}_{\mathbf{f}(\mathbf{p})}, \mathbf{y}_{\mathbf{f}(\mathbf{p})}, \mathbf{z}_{\mathbf{f}(\mathbf{p})})$ are basis vectors for $\mathbf{p}$ and $\mathbf{f}(\mathbf{p})$, and $(x_{\mathbf{p}}, y_{\mathbf{p}}, z_{\mathbf{p}})$ and $(x_{\mathbf{f}(\mathbf{p})}, y_{\mathbf{f}(\mathbf{p})}, z_{\mathbf{f}(\mathbf{p})})$ are 3D positions of $\mathbf{p}$ and $\mathbf{f}(\mathbf{p})$ in global coordinate system, respectively. In this study, $\mathbf{z}_{\mathbf{p}}$ is determined as the unit normal vector of the point $\mathbf{p}$. $\mathbf{x}_{\mathbf{p}}$ and $\mathbf{y}_{\mathbf{p}}$ is calculated as Eqs. (5) and (6).

$$\mathbf{x}_{\mathbf{p}} = \frac{(0,1,0)^t \times \mathbf{z}_{\mathbf{p}}}{\| (0,1,0)^t \times \mathbf{z}_{\mathbf{p}} \|}, \quad (5)$$

$$\mathbf{y}_{\mathbf{p}} = \mathbf{z}_{\mathbf{p}} \times \mathbf{x}_{\mathbf{p}}. \quad (6)$$

$\mathbf{x}_{\mathbf{f}(\mathbf{p})}$, $\mathbf{y}_{\mathbf{f}(\mathbf{p})}$ and $\mathbf{z}_{\mathbf{f}(\mathbf{p})}$ are also determined in the same way.

2.2. Energy minimization: Update of positions of points

After generating initial points and faces in missing regions, positions of points are updated so as to minimize the energy function E defined in Eq. (1) using a framework of greedy algorithm. In our definition of the energy E, the energy for each point can be treated independently if similar shape pairs $(\mathbf{p}, \mathbf{f}(\mathbf{p}))$ determined by Eq. (2) can be fixed. Thus, the following two processes are repeated until the energy converges: (i) update of similar shape pairs fixing positions of points, and (ii) parallel update of all the positions of points fixing similar shape pairs.

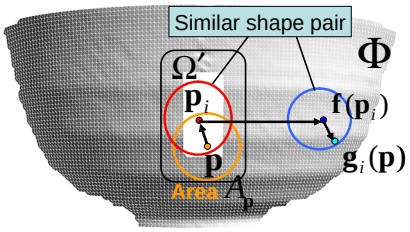


Fig. 4. Relationship between points in energy calculation.

In the process (i) above, the update of the similar shape pair is performed by calculating $\mathbf{f}(\mathbf{p})$ fixing the positions of all the points in the missing region with Eq. (2). Concretely, SSD is calculated by Eq. (3) for all the points in the data region Φ and the point which gives a minimum SSD value is determined as the most similar point $\mathbf{f}(\mathbf{p})$. In the process (ii), the positions of all the points $\mathbf{p}$ in the missing region are updated in parallel so as to minimize the energy defined by Eq. (1). In the following, we describe the method for calculating the positions of points $\mathbf{p}$ for fixed similar shape pairs. First, the energy E is resolved into the element energy $E(\mathbf{p})$ for each point in the missing region. As shown in Fig. 4, the target point to be updated is $\mathbf{p}$, and the position of the point inside a spherical area $A_\mathbf{p}$ can be expressed as $\mathbf{p}_i$ and is corresponded to $\mathbf{f}(\mathbf{p}_i)$ by Eq. (2). In this case, the point corresponding to the point $\mathbf{p}$ is $\mathbf{g}_i(\mathbf{p})$. Now, the element energy $E(\mathbf{p})$ can be defined in terms of $\mathbf{p}$, $\mathbf{g}_i(\mathbf{p})$ and matrix $\mathbf{M}_{\mathbf{f}(\mathbf{p}_i)\mathbf{p}_i}$ for surface alignment as follows:

$$E(\mathbf{p}) = \sum_{\mathbf{p}_i \in A_\mathbf{p}} \frac{w_{\mathbf{p}_i}}{N(A_\mathbf{p})} \parallel \mathbf{p} - \mathbf{M}_{\mathbf{f}(\mathbf{p}_i)\mathbf{p}_i}\mathbf{g}_i(\mathbf{p}) \parallel^2 . \quad (7)$$

The relationship between the total energy E and the element energy $E(\mathbf{p})$ for each point can be written as follows:

$$E = \sum_{\mathbf{p}_k \in \Omega} E(\mathbf{p}_k) + C. \quad (8)$$

C is the energy for the points in the region $\overline{\Omega} \cap \Omega'$, and is treated as a constant because positions of points and all the similar shape pairs are fixed in this region in the process (ii).

Here, note that all the corresponding points $\mathbf{M}_{\mathbf{f}(\mathbf{p}_i)\mathbf{p}_i}\mathbf{g}_i(\mathbf{p})$ ($\forall \mathbf{p}_i \in A_\mathbf{p}$) exist in the normal direction of point $\mathbf{p}$. Thus, as shown in Fig. 5, points $\mathbf{p}$ and $\mathbf{M}_{\mathbf{f}(\mathbf{p}_i)\mathbf{p}_i}\mathbf{g}_i(\mathbf{p})$ can be represented by using the normal vector $\mathbf{n}_\mathbf{p}$ of $\mathbf{p}$, an arbitrary point $\mathbf{p}_0$ in the normal direction of $\mathbf{p}$ and parameters $t_\mathbf{p}$ and $t_{(\mathbf{p}_i,\mathbf{p})}$ as follows:

$$\mathbf{p} = \mathbf{p}_0 + t_\mathbf{p}\mathbf{n}_\mathbf{p}, \quad (9)$$

$$\mathbf{M}_{\mathbf{f}(\mathbf{p}_i)\mathbf{p}_i}\mathbf{g}_i(\mathbf{p}) = \mathbf{p}_0 + t_{(\mathbf{p}_i,\mathbf{p})}\mathbf{n}_\mathbf{p}. \quad (10)$$

By substituting them into Eq. (7), the element energy is represented as follows:

$$E(\mathbf{p}) = \sum_{\mathbf{p}_i \in A_\mathbf{p}} \frac{w_{\mathbf{p}_i}}{N(A_\mathbf{p})} (t_\mathbf{p} - t_{(\mathbf{p}_i,\mathbf{p})})^2. \quad (11)$$

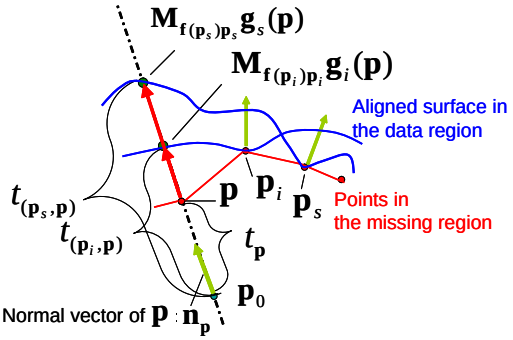


Fig. 5. Conversion of parameters.

Here, by the assumption that the normal vector $\mathbf{n}_\mathbf{p}$ does not change after updating the position of $\mathbf{p}$, all the parameters in $E(\mathbf{p})$ except $t_\mathbf{p}$ are constants. In addition, the change of $t_\mathbf{p}$ dose not affect the element energy of other points. Thus, by minimizing the element energy $E(\mathbf{p})$ independently, whole energy E can be minimized. The parameter $t_\mathbf{p}$ that minimizes $E(\mathbf{p})$ is calculated as follows:

$$t_\mathbf{p} = \frac{\sum_{\mathbf{p}_i \in A_\mathbf{p}} w_{\mathbf{p}_i} t_{(\mathbf{p}_i,\mathbf{p})}}{\sum_{\mathbf{p}_i \in A_\mathbf{p}} w_{\mathbf{p}_i}}. \quad (12)$$

From Eqs. (9), (10) and (12), the position of $\mathbf{p}$ that minimizes $E(\mathbf{p})$ is calculated as follows:

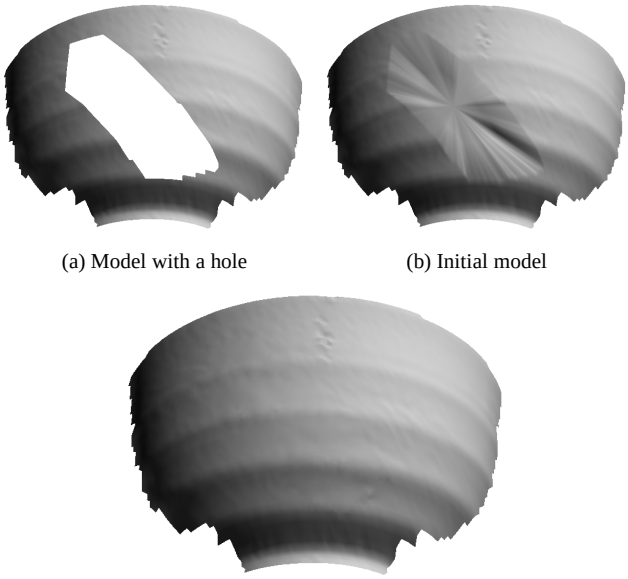
$$\mathbf{p} = \frac{\sum_{\mathbf{p}_i \in A_\mathbf{p}} w_{\mathbf{p}_i} \mathbf{M}_{\mathbf{f}(\mathbf{p}_i)\mathbf{p}_i}\mathbf{g}_i(\mathbf{p})}{\sum_{\mathbf{p}_i \in A_\mathbf{p}} w_{\mathbf{p}_i}}. \quad (13)$$

Actually, the value in Eq. (13) is an approximate solution because the normal vector $\mathbf{n}_\mathbf{p}$ changes due to the update of the positions of $\mathbf{p}$. However, a good solution can be obtained as the energy converges because the change of the normal vector becomes smaller as the energy converges.

3. EXPERIMENT

To demonstrate the effectiveness of the proposed method, we have applied the method to two surface models. They have different characteristics as shown in Figs. 6(a) and 7(a). The missing region of each model were given manually. In the experiment, as the initial points in the missing region, the gravity point of the boundary points of the missing region and the median points between the gravity point and each boundary point were given. Faces were generated so as to connect these points.

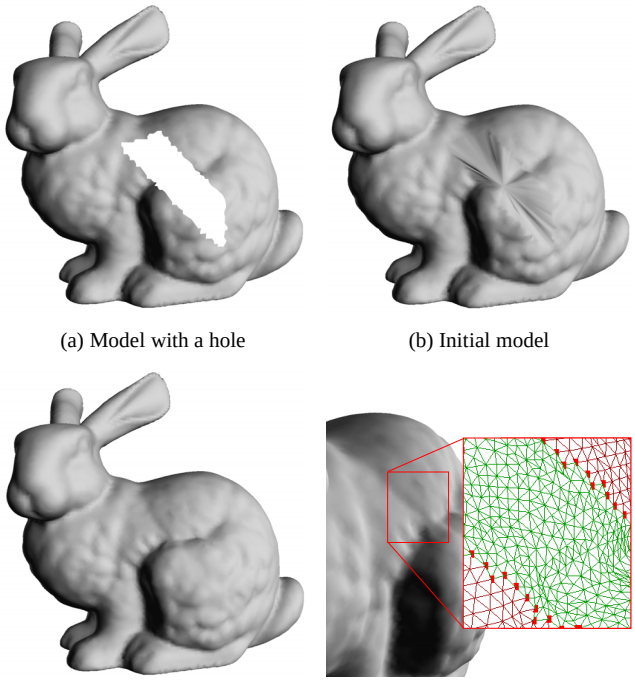
Model A is a bowl model that has a smooth curved surface and raised edges along the curve around the missing region. Fig. 6(b) shows the model with initial points and faces. The initial model is not natural due to rectilinear faces. By refining this initial model with our method, as illustrated in Fig. 6(c), the missing region is completed with a smooth curved surface similar to the surface in the data region. Raised edges are



(a) Model with a hole (b) Initial model

(c) Completed model

Fig. 6. Completion for a bowl with a hole (Model A).



(a) Model with a hole (b) Initial model

(c) Completed model (d) Completed solid and mesh model from a different angle

Fig. 7. Completion for Bunny with a hole (Model B).

also reconstructed and totally the completed model looks very natural.

Model B is Stanford Bunny that has a rugged surface and a dent edge between the body and the leg of the bunny around the missing region. By refining the surface from the initial model shown in Fig. 7(b), a natural rugged surface is generated and a dent edge is connected in the missing region as shown in Fig. 7(c). The completed solid and mesh model from a different angle is shown in Fig. 7(d). Mesh arrangement looks also natural on the rugged face and the dent edge.

4. CONCLUSION

In this paper, we have presented a novel method for surface completion based on energy minimization. In the experiment for two kinds of shape objects, our method have generated the complex and natural surface shapes in the missing regions. In future work, our method will be applied to 3D models for large environments such as outdoor scenes. A surface completion method for shape and color should also be investigated.

Acknowledgement

This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Scientific Research (A), 19200016, and Young Scientists (B), 20700162.

5. REFERENCES

[1] J. Verdera, V. Caselles, M. Bertalmio, and G. Sapiro, "Inpainting Surface Holes," in *Proc. Int, Conf. on Image Processing*, 2003, vol. 2, pp. 903–906.

[2] J. Wang and M.M. Oliveira, "A Hole-Filling Strategy for Reconstruction of Smooth Surfaces in Range Images," in *Proc. SIBGRAPI*, 2003, pp. 11–18.

[3] H. Xie, K.T. McDonnell, and H. Qin, "A Finite Element Method for Surface Restoration with Smooth Boundary Conditions," *Computer Aided Geometric Design*, vol. 21, No. 5, pp. 427–445, 2004.

[4] J. Davis, S.R. Marschner, M. Garr, and M. Levoy, "Filling Holes in Complex Surfaces Using Volumetric Diffusion," in *Proc. Int. Symp. on 3D Data Processing, Visualization and Transmission*, 2002, pp. 428–438.

[5] J. Podolak and S. Rusinkiewicz, "Atomic Volumes for Mesh Completion," in *Proc. Eurographics Symp. on Geometry Processing*, 2005, pp. 33–41.

[6] A. Sharf, M. Alexa, and D. Cohen-Or, "Context-based Surface Completion," in *Proc. ACM SIGGRAPH*, 2004, pp. 878–887.

[7] M. Pauly, "Example-Based 3D Scan Completion," in *Proc. Eurographics Symp. on Geometry Processing*, 2005, pp. 23–32.

[8] V. Kraevoy and A. Sheffer, "Template-Based Mesh Completion," in *Proc. Eurographics Symp. on Geometry Processing*, 2005, pp. 13–22.

[9] T.P Breckon and R. B. Fisher, "Plausible 3D Colour Surface Completion Using Non-parametric Techniques," in *Proc. Int. Conf. on the Mathematics of Surfaces*, 2005, pp. 102 – 120.

[10] S. Park, X. Guo, H. Shin, and H. Qin, "Surface Completion for Shape and Appearance," *Int. J. of Computer Graphics*, vol. 22, No. 3, pp. 168 – 180, 2006.