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Effect of Weak Localization on Microwave Conductivity in Two-Dimensional Superconductors at Absolute Zero

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The weak localization correction for the linear absorption spectrum of the two-dimensional s-wave superconductor is calculated at absolute zero. This correction term does not diverge even in a two-dimensional system, and its dependence on frequency is similar to that of Mattis-Bardeen's conductivity. The effect of the Coulomb interaction coupled with superconducting fluctuations mainly contributes to the correction term. The excitation energy of the collective mode due to this coupling remains low in a two-dimensional system, and it is proportional to the square root of the wavenumber. The dispersion of this excitation mode is related to the absolute value of the interaction vertex, and therefore affects the magnitude of the weak localization correction terms.

KEYWORDS: weak localization correction, Goldstone mode, conductivity, dirty superconductors, vertex correction

1. Introduction

The weak localization effect originates from a coherent backscattering by impurities and the change of Coulomb interaction between electrons owing to a diffusive motion of electrons. This effect has been studied especially in the normal state, and weak localization correction terms for conductivity have been calculated [1–3]. Similar effects exist in the superconducting state, and there are calculations of the superfluid density [4] and the thermal conductivity [5] with the effect of backscattering by impurities included.

Recently, the weak localization correction term was calculated for the conductivity in the superconducting state owing to the change in Coulomb interaction between electrons in the presence of impurity scatterings. The correction term owing to the Coulomb interaction combined with the superconducting fluctuation is shown to be larger than that caused by the coherent backscattering. This calculation provides a correction term for Mattis-Bardeen's formula [6] for conductivity in the superconducting state, and reveals that the weak localization correction to thermal excitations is smaller than that to excitations across the superconducting gap [7].

Since this calculation is performed in a three-dimensional system, the phase fluctuation is pushed up to the plasma frequency [8]. On the other hand, the low-energy phase fluctuation has been considered as an interpretation [9] of the experimental results for the conductivity in dirty superconductors. Therefore, in this study, we calculate the weak localization correction effect in a two-dimensional system in which the phase fluctuation remains at low energy. In this case it is important to take account of the fact that the dispersion of the phase fluctuation is related to the magnitude of the interaction vertex.

2. Correction Terms for Conductivity

The derivation of the weak localization correction terms for conductivity in the three-dimensional system is given in Ref. [7]. The diagrams of these correction terms are shown in Fig. 1. These dia-

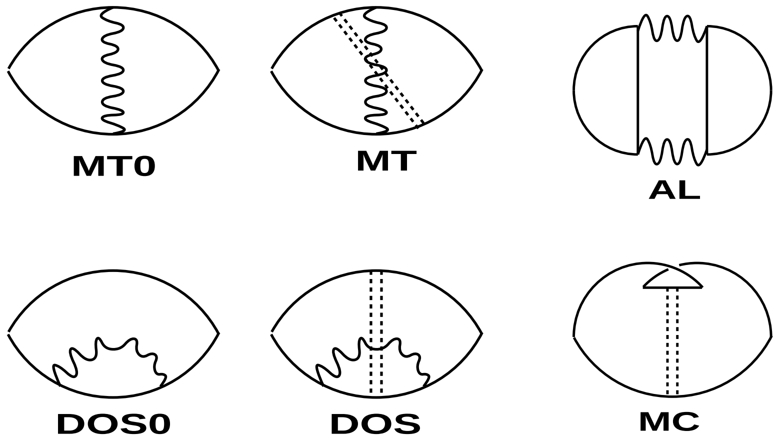

Fig. 1. The diagrams for the weak localization correction terms. "MT", "DOS", "AL", and "MC" mean Maki-Thompson, density of states, Aslamazov-Larkin, and maximally crossed terms, respectively. "0" in "MT0" and "DOS0" indicates that the ladder of impurity scattering (a double dashed line) is absent. The wavy line means the fluctuation effect, which includes the superconducting fluctuation and the long-range Coulomb interaction.

grams have been used to investigate the superconducting fluctuation phenomena [10–12]. The absorptive part of the conductivity (divided by a conductivity in the normal state σ_0) is given by

$$\mathrm{Re}\sigma_\omega/\sigma_0 = (\mathrm{Re}\sigma_\omega^{(0)} + \mathrm{Re}\sigma_\omega^{\mathrm{vc}})/\sigma_0 \tag{1}$$

with $\mathrm{Re}\sigma_\omega^{(0)}$ being usual Mattis-Bardeen term [6] and

$$\mathrm{Re}\sigma_\omega^{\mathrm{vc}} = \mathrm{Re}\left(\sigma_\omega^{MT0} + \sigma_\omega^{MT} + \sigma_\omega^{DOS0} + \sigma_\omega^{DOS} + \sigma_\omega^{AL} + \sigma_\omega^{MC}\right). \tag{2}$$

The expressions for the weak localization correction terms in the two-dimensional system are similar to those in the three-dimensional system which are given in Ref. [7]. Here, we mention different points only.

$$\frac{\mathrm{Re}\sigma_\omega^{\mathrm{xx}}}{\sigma_0} = \frac{1}{4\pi\omega k_F l}\int dx \int d\epsilon \begin{cases} -\int d\omega' \mathrm{Im} Q^{MT0}_{\epsilon,\omega',x}(\omega)/2\pi & (\mathrm{xx} = MT0) \\ -\int d\omega' \mathrm{Im} Q^{DOS0}_{\epsilon,\omega',x}(\omega)/8\pi & (\mathrm{xx} = DOS0) \\ \mathrm{Re}\sum_{s=\pm} s\mathrm{Tr}[\hat{\tau}_0 + \hat{g}^+_{\epsilon+\omega}\hat{g}^s_\epsilon]/(x + \zeta^+_{\epsilon+\omega} + \zeta^s_\epsilon)\Big|_{-\omega<\epsilon<0} & (\mathrm{xx} = MC) \end{cases} \tag{3}$$

for MT0, DOS0, and MC. (We set $\hbar = c = 1$.) $x = Dq^2$ with $D = v_F^2\tau/2$ being the diffusion constant (v_F and τ are the Fermi velocity and the elastic relaxation time), and k_F and $l = v_F\tau$ are the Fermi wave number and mean free path, respectively. $\hat{g}^{+(-)}_\epsilon$ is the retarded (advanced) quasiclassical Green function written in the Nambu representation ($\hat{\tau}_0$ is the unit matrix), and $\zeta^\pm_\epsilon = -i\mathrm{sgn}(\epsilon)\sqrt{\epsilon^2 - \Delta^2}\theta(|\epsilon| - \Delta) + \sqrt{\Delta^2 - \epsilon^2}\theta(\Delta - |\epsilon|)$ with Δ being the superconducting gap.

$$\frac{\mathrm{Re}\sigma_\omega^{\mathrm{xx}}}{\sigma_0} = \frac{1}{2\pi^2\omega k_F l}\int dx \int d\omega' \begin{cases} x\int d\epsilon \mathrm{Im} Q^{MT}_{\epsilon,\omega',x}(\omega)/8 & (\mathrm{xx} = MT) \\ -x\int d\epsilon \mathrm{Im} Q^{DOS}_{\epsilon,\omega',x}(\omega)/8 & (\mathrm{xx} = DOS) \\ x\mathrm{Re} Q^{AL}_{\omega',x}(\omega)/\pi & (\mathrm{xx} = AL) \end{cases} \tag{4}$$

for MT, DOS, and AL. There are two different points between the two- and three-dimensional systems. The correction term is proportional to $1/k_F l$ in the former system, and there is a difference in dependences of the integrand on $x = Dq^2$ (q is wave number). $Q^{\mathrm{xx}}_{(\epsilon,)\omega',x}(\omega)$ for xx $= MT0$, MT, $DOS0$, DOS, and AL are the same as those given in Ref. [7], and we do not rewrite these quantities here.

3. Goldstone Mode

The interaction vertices representing the superconducting phase fluctuation [$\Gamma_2(q)$], the screened Coulomb interaction [$\Gamma_3(q)$], and the coupling of these effects [$\Gamma_4(q)$] are given by the following equations.

$$\begin{pmatrix} \Gamma_3(q) \\ \Gamma_2(q) \\ \Gamma_4(q) \end{pmatrix} \simeq \frac{1}{1/p + \chi_2 + c_q[-\chi_3(1/p+\chi_2) + (2\chi')^2]} \begin{pmatrix} c_q(1/p+\chi_2)/2 \\ -(1-c_q\chi_3)/2 \\ -c_q\chi' \end{pmatrix}. \tag{5}$$

The subscripts 2 and 3 are related to those of Pauli matrices, which represent the phase of the order parameter and the density of electrons, respectively. Goldstone mode in the two-dimensional system is obtained by calculating the denominator of $\Gamma_{3,2,4}(q)$. Here, $c_q = (\pi/2)\rho_0 v_q^C$ ($\rho_0 = m/\pi$ and m is the mass of quasiparticles) with $v_q^C = 2\pi e^2/q$ being the Fourier transform of the Coulomb interaction in the two-dimensional system, and p is the dimensionless electron–phonon coupling constant with the weak-coupling approximation. The definitions of χ_2, χ_3, and χ' for the two-dimensional system are the same as those for the three-dimensional system given in Ref. [7]. The denominator of $\Gamma_{3,2,4}(q)$ is written as

$$\frac{1}{p} + \chi_2 + c_q\left[-\chi_3\left(\frac{1}{p}+\chi_2\right) + (2\chi')^2\right] \simeq \frac{-2}{\pi}\left(\frac{\omega}{2\Delta}\right)^2 + \frac{c_q D q^2}{\pi\Delta} - i\mathrm{sgn}(\omega)0^+ \tag{6}$$

for small ω and q. Then, the dispersion relation of Goldstone mode in dirty two-dimensional superconductors is given by

$$\omega_q = \sqrt{\pi\Delta\tau}\sqrt{\frac{2\pi e^2 n_e}{m}|q|} \tag{7}$$

($n_e = k_F^2/2\pi$). The relation $\omega \propto q^{1/2}$ is similar to that for Goldstone mode in the clean case $\sqrt{2\pi e^2 n_e |q|/m}$ [13, 14]. In the dirty case, however, there is an extra factor $\sqrt{\pi\Delta\tau}$ which depends on impurity scattering. (In the three-dimensional system $v_q^C = 4\pi e^2/q^2$ ($\rho_0 = mk_F/\pi^2$), and then $\sqrt{2\Delta c_q D q^2}$ becomes a constant plasma frequency for small q as in Ref. [8].)

3.1 Fluctuations at finite temperatures

Here, we consider the case at finite temperatures. The dispersion of Goldstone mode in the neutral system is given by $1/p + \chi_2 = 0$, and its excitation energy is proportional to q as $\omega_q = q\sqrt{\pi\Delta D}$ [15]. At finite temperatures ($T \neq 0$) the contribution from the superconducting fluctuation is written as $\sum_{\boldsymbol{q}} \int d\omega \coth\left(\frac{\omega}{2T}\right) \mathrm{Im}\Gamma_2(q)$. In the neutral system

$$\mathrm{Im}\Gamma_2(q) \simeq -\pi^2\Delta^2 \mathrm{sgn}(\omega)\delta(\omega^2 - \pi\Delta D q^2), \tag{8}$$

and then

$$\int \frac{qdq}{2\pi} \int d\omega \coth\left(\frac{\omega}{2T}\right) \mathrm{Im}\Gamma_2(q) \simeq \int dq \frac{-T\Delta}{Dq}, \tag{9}$$

which results in an infrared logarithmic divergence [16, 17]. In the charged system

$$\mathrm{Im}\Gamma_2(q) \simeq -2\pi\Delta^2 c_q \mathrm{sgn}(\omega)\delta(\omega^2 - 2\Delta c_q D q^2) \tag{10}$$

from Eqs. (5) and (6), and

$$\int \frac{qdq}{2\pi} \int d\omega \coth\left(\frac{\omega}{2T}\right) \mathrm{Im}\Gamma_2(q) \simeq \int dq \frac{-c_q T\Delta}{c_q D q}. \tag{11}$$

Therefore, c_q cancels out from the integrand, and Eq. (11) diverges in the same way as Eq. (9) though the q-dependence of Goldstone mode in the charged system ($\omega_q \propto \sqrt{q}$) is different from that in the neutral system ($\omega_q \propto q$). The numerical calculation in the next section is performed at $T = 0$ and the above divergence does not occur.

4. Numerical Calculations

The numerical calculation can be performed as in Ref. [7], but in the two-dimensional case the assumption $\left|c_q[-\chi_3(1/p+\chi_2)+(2\chi')^2]\right| \gg \Delta$ cannot be used because the quantity on the left-hand side is proportional to $|q|$ for small q. Thus, in addition to $k_F l$, a material-dependent dimensionless parameter,

$$s_g := \frac{\pi}{2}\sqrt{\frac{k_F\xi_0}{n_e a_B^2}} \tag{12}$$

($\xi_0 = v_F/\pi\Delta$ is the coherence length and $a_B = 1/me^2$), is introduced in numerical calculations. With use of this quantity Eq. (7) is rewritten as $\omega_q = \left(2s_g\Delta\sqrt{xk_F l\Delta}\right)^{1/2}$, and $c_q = s_g\sqrt{k_F l\Delta/x}$.

The real and imaginary parts of Eq. (5) are shown in Fig. 2. In the case of $s_g = 5.0$ a peak

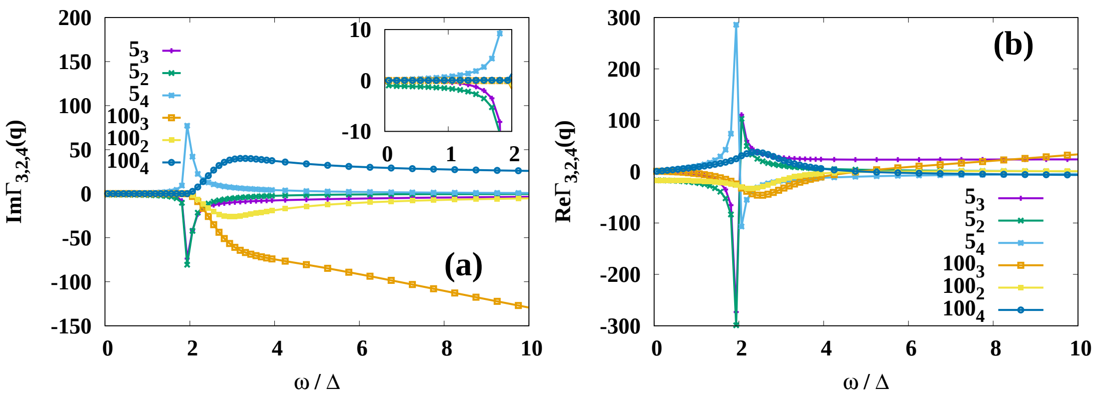

Fig. 2. Dependences of $\Gamma_{3,2,4}(q)$ on ω at $x = Dq^2 = 0.0625$. (a) Imaginary parts of $\Gamma_{3,2,4}(q)$. (The inset shows the same results for the range $0 < \omega/\Delta < 2$.) (b) Real parts of $\Gamma_{3,2,4}(q)$. The values of n_m on the left of lines mean that $\Gamma_m(q)$ with $s_g = n$. $k_F l = 6.0$.

originated from the Goldstone mode is visible around $\omega \simeq 2\Delta$ for $x = Dq^2 = 0.0625$. In the case of $s_g = 100.0$ the value of x should be smaller than 0.00016 for the appearance of the Goldstone mode around $\omega \simeq 2\Delta$. Then, the observation of Goldstone mode is difficult in the case of the weak-coupling superconductors ($s_g \propto \sqrt{k_F\xi_0} \gg 1$).

The calculated results of the optical conductivity are shown in Fig. 3. The vertex corrections suppress the optical conductivity regardless of the values of s_g and $k_F l$ as in Ref. [7]. As compared to the three-dimensional system [7], the suppression by vertex corrections is large in the two-dimensional system, because of the coefficient $1/k_F l$ in Eqs. (3) and (4). The difference between calculated results for $s_g = 5.0$ and $s_g = 100.0$ originates from a difference in the effective interactions $\Gamma_{3,2,4}(q)$. This

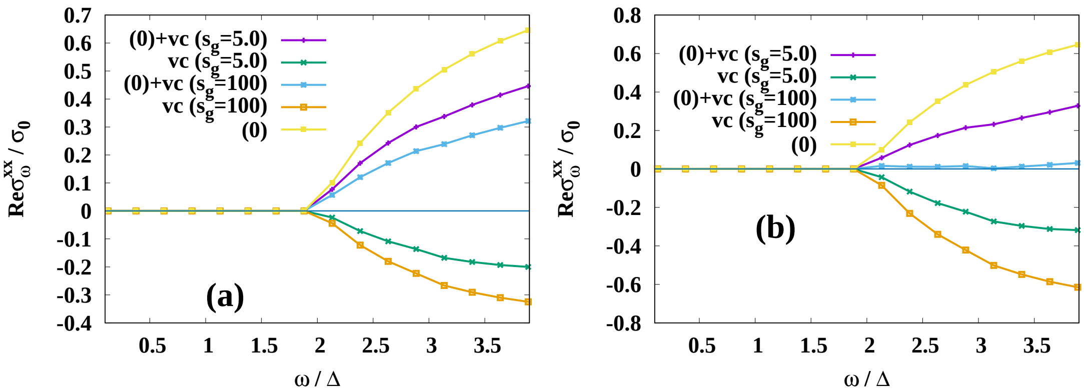

Fig. 3. Dependences of optical conductivity on ω for $s_g = 5.0$ and $s_g = 100.0$. "(0)+vc", "vc", and "(0)" indicate that $\sigma^{\rm xx}_\omega = \sigma^{(0)}_\omega + \sigma^{\rm vc}_\omega$, $\sigma^{\rm vc}_\omega$, and $\sigma^{(0)}_\omega$, respectively. (a) $k_F l = 10.0$. (b) $k_F l = 6.0$.

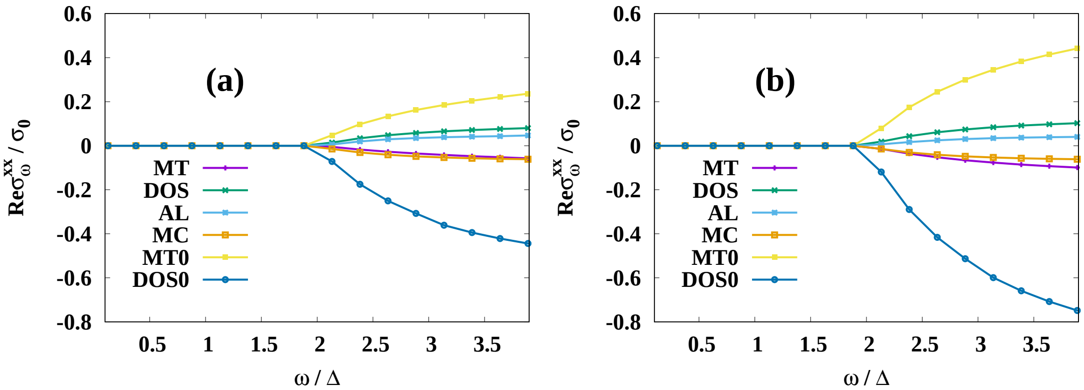

Fig. 4. Dependences of vertex correction terms on ω for $k_F l = 10.0$. "MT", "DOS", "AL", "MC", "MT0", and "DOS0" indicate that $\sigma^{\rm xx}_\omega = \sigma^{MT}_\omega$, σ^{DOS}_ω, σ^{AL}_ω, σ^{MC}_ω, σ^{MT0}_ω, and σ^{DOS0}_ω, respectively. (a) $s_g = 5.0$. (b) $s_g = 100.0$.

difference is shown in Fig. 4. Reσ^{MC}_ω does not depend on s_g and takes same values in Figs. 4(a) and 4(b). The vertex correction terms take the same signs and similar relative magnitudes between them as those in the three-dimensional system [7]. ("MC" and "DOS0+DOS" terms decrease the absorption spectrum due to the coherent backscattering and the suppression of the density of states, respectively. On the other hand, the signs of "MT0+MT" and "AL" terms indicate that the superconducting fluctuation increases the absorptive part of conductivity.) A comparison between $s_g = 5.0$ and $s_g = 100.0$ indicates that the dependences of $\Gamma_{3,2,4}(q)$ on s_g result in a difference of the weak localization corrections between strong- and weak-coupling superconductors.

5. Discussion

In the two-dimensional system there exists Goldstone mode and the imaginary parts of vertices Im$\Gamma_{3,2,4}(q)$ take finite values for $\omega < 2\Delta$ in contrast to the three-dimensional system [18]. This low-energy mode, however, does not induce the finite absorption below 2Δ as numerically shown in the previous section (Re$\sigma_{\omega<2\Delta} = 0$). This can be confirmed also by considering the analytical expression Eqs. (3) and (4). This consequence is different from that in the case of inhomogeneous (granular) superconductors in which Re$\sigma_{\omega<2\Delta} \neq 0$ [9].

The contribution of the Goldstone mode to the weak localization correction is small except for the case of strong-coupling superconductors. This is because the effective range of Goldstone mode is very narrow (in terms of its frequency and wave number) as compared to the range of integrations $\sim 1/\tau$ ($\gg \Delta$) in Eqs. (3) and (4). When the Goldstone mode is effective in the wide range of frequencies and wave numbers, the vertices $|\Gamma_{3,2,4}(q)|$ take small values. Thus, in strong-coupling superconductors, the weak localization correction to the linear absorption is small as compared to weak-coupling superconductors. The Goldstone mode seemingly enhances the absorption spectrum, and keeps the system away from the localization transition.

The numerical computation in this work was carried out at the Yukawa Institute Computer Facility.

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