Doctoral Dissertation

Modelling Collective Behaviour in Shepherding and Horse Herding

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Abstract

Many species in the animal kingdom have evolved to live and travel together in groups called herds. They perform herding behaviours that would be advantageous to them in certain ecological conditions. In most groups, there are herders and followers. Here, we present two models of collective behaviour from the perspective of the herder and followers in two different animal groups.

First, we consider the herding of sheep by a shepherd dog. We develop a model on how a herder might have acquired such a herding behaviour using imitation learning. We show that given expert demonstrations from a phenomenological model, our imitation learning model reproduces the switching behaviour necessary to be able to complete the herding task. Furthermore, our model does simply copy expert demonstrations but is able to generalise well to herd more sheep than those in the given demonstrations.

Second, we model the behaviour of female horses as a response to herding by a harem stallion. We propose a mathematical model, a modification of the phenomenological model, where the horses' motion is expressed as a sum of a linear combination of forces. We then optimise the coefficients of the forces based on actual data. Results show that our model is able to recreate the trajectories and directional trends of the original data set.

Finally, we discuss the relevance and implications of these models in animal behaviour studies.

Keywords:

imitation learning, herding, animal behaviour, modelling, optimisation

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1 Introduction

1.1 Background

Many species in the animal kingdom naturally live and travel together in groups called herds [1]. The practise of herding is an example of collective behaviour that arises for different reasons: be it for keeping group cohesion [2, 3], evading a predator [4–6], or for finding a common resource, among others. For the purposes of this research, we define herding as directing a group of animals from place to place in a unified direction.

In most social animal groups, individuals are divided into two groups according to the amount of influence they wield [7–9]. What constitutes a leader of a group is still the subject of many ongoing studies [10,11] in animal behaviour research. For the purposes of this dissertation, we define members that initiate group motion to be the herder. Other members which move as a response to the initial motion are called followers. In the case of a shepherd dog herding a flock of sheep, the shepherd dog is the herder while the sheep are the followers. In a horse harem, on the other hand, the stallion is initiates the motion of the group and is therefore the herder. The mares that subsequently react to the stallion's motion are the followers.

Over time, animals have naturally evolved to live together and perform herding behaviours that would be advantageous to them under certain ecological conditions [1]. This is especially true in the case of horses living in the wild, where herding of mares is primarily performed by the stallion [12–15]. Shepherd dogs, on the contrary, can be trained by humans to herd livestock [16–18].

A wide variety of methods in modelling and analysing collective behaviour abound in animal behaviour literature. In this dissertation, we closely follow a phenomenological model of herding of sheep by a shepherd dog [19]. Individual sheep outside a group are prone to become prey. When a predator is detected, individuals aggregate to form a big cohesive group to prevent predation. This mechanism is famously known as the *selfish herd theory* [5, 6].

In the phenomenological model, the motion of individual sheep is governed by a linear combination of forces. These results will play a central role in our work, where we will adapt their results for a better understanding of herding behaviour in two animals species: sheep and wild horses.

In this dissertation, we present two studies on collective behaviour. The first part focuses on the herder. Since herding is a complex task, we discuss how a shepherd dog might be able to learn the strategies that are necessary to complete the herding task by proposing an imitation learning framework.

Next, we focus on the mares in a horse harem. Here we modify the phenomenological model to express the motion of the mares as a linear combination of different forces. Together with the computed forces and the measured velocities from field observations, we then create a mathematical model to describe the motion of mares while being herded by a stallion.

Our results in these studies will enable us to understand collective behaviour from the point of view of both the herder and the followers.

1.2 Related Works

Also central to this thesis are the concepts of reinforcement learning (RL) and imitation learning (IL). Reinforcement learning is learning by an agent and its interaction with the environment [20]. Imitation learning, on the other hand, is a learning model where a learner watches the demonstrations performed by an expert [21–25]. These techniques are increasing popular in the field of animal behaviour research.

1.3 Organisation of the Thesis

The rest of the dissertation is organised in the following manner: Chapter 2 discusses the phenomenological model of herding from which both our studies are based on. Chapter 3, then, focuses on the herder. Here we discuss the herding of sheep by a shepherd dog using imitation learning techniques. Chapter 4 focuses on the followers. We propose a mathematical model to describe the behaviour of female horses as a response to herding by a stallion. Finally, in Chapter 5 we summarise our important results, explore their possible implications in animal behaviour studies, and provide recommendations for future studies.

2 Phenomenological Model of Herding of Sheep by a Shepherd Dog

This study is largely based on the phenomenological model of herding developed by Strombom, et al [19]. In this section, we highlight the main contributions of the model.

We consider an unobstructed field with dimensions of 300m $\times$ 300m. For simplicity, we consider a Cartesian coordinate system in the domain $[0, 300] \times [0, 300]$. Sheep are initialised with a uniform distribution in the region $[75 \times 150] \times [75, 150]$ and are free to move beyond this region. The task of the shepherd dog is to be able to bring the entire flock of sheep to a predetermined target location. Herding ends successfully when the centre of mass of the flock is within a small distance from the target position.

2.1 Sheep Model

We let S be the position of the shepherd dog and A_i be the position of the ith sheep. The motion of each sheep is governed by interaction rules composed of a linear combination of five forces [19] enumerated as follows:

1. An inertial force, H_i, which attracts each sheep to its previous movement or direction.

2. An attractive force, C_i, towards the centre of mass (COM) of the group, which is given by

$$C_i = \mathrm{COM} - A_i. \tag{2.1}$$

This long-range attractive force is based on the selfish herd theory [5, 6], where prey aggregate to form a compact group in the face of a threat.

3. A short-range repulsive force, $\hat{R}_i^A$,

$$\hat{R}_i^A = \sum_{i \neq j} \frac{A_i - A_j}{|A_i - A_j|}, \tag{2.2}$$

so that sheep maintain a distance from each other during the motion.

4. A repulsive force, R_i^S, away from the herder, which is given by

$$R_i^S = A_i - S, \tag{2.3}$$

when the herder is within a certain distance from the sheep.

5. A small random error term, ϵ_i.

The new position of sheep i is then given by

$$A_i' = A_i + \delta \hat{H}_i, \tag{2.4}$$

where δ is the step size of the sheep and

$$H_i = h\hat{H}_i + c\hat{C}_i + \rho_A \hat{R}_i^A + \rho_S \hat{R}_i^S + e\epsilon_i, \tag{2.5}$$

$$\hat{H}_i = H_i / |H_i|. \tag{2.6}$$

The coefficients h, c, ρ_A, ρ_S, and e, which refer to the strengths of their corresponding forces, are heuristically chosen.

2.2 Dog Model

The motion of the dog consists of two strategies: collecting and driving, depending on the state of the flock of sheep. When the distance of the furthest sheep from the COM of a flock of N sheep is more than a threshold $r = 2N^{2/3}$, the dog proceeds to point P_c to collect the furthest sheep. Here, point P_c is located on the extended line connecting the COM to the furthest sheep and two units distant from the furthest sheep (Fig. 2.1b).

Otherwise, the dog proceeds to point P_d to drive the flock. Here, point P_d is located on the extended line connecting the final goal to the COM and $2\sqrt{N}$ units distant from the COM (Fig. 2.1b).

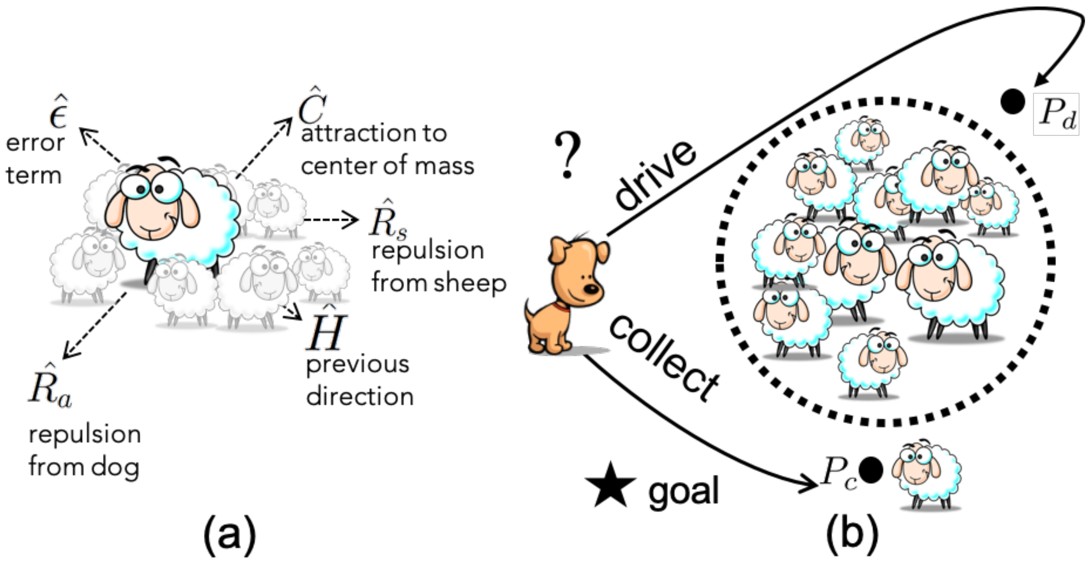

Figure 2.1: (a) Sheep model governed by a linear combination of five forces. (b) Dog model consisting of two strategies, collecting and driving.

The speed of the shepherd dog is constant for each strategy. The dog moves 1.5 times as fast as each sheep during collecting and 0.6 times as fast during driving so that no sheep can outrun the dog.

3 Herding of Sheep by a Shepherd Dog

3.1 Introduction

In this chapter, we focus on the herder. In animal groups, we define the herder to be the initiator of motion. Here, we focus on the shepherd dog (*Canis familiaris*) as the herder while it performs herding on a flock of sheep to a predetermined location. The task of herding is complex. There is a risk of divergence at each time-step during the herding task, when the flock uncontrollably dissipates and the herding task is unsuccessfully performed.

Animals have naturally evolved to perform herding in the wild [1]. Shepherd dogs, on the other hand, are trained by humans with a series of commands and reinforcements to be able to learn the task. Thus, it is interesting to construct a learning model to possibly explain how shepherd dogs are able to acquire such behaviour [26]. Although this kind of learning is generally modelled using reinforcement learning [27], the task of shepherding is so complex that no mathematical models for learning it have succeeded in reproducing the switching policy to the best of our knowledge. In reinforcement learning, a reward is only given when the task is completed, which is not practical during herding. Some models of learning in herding exist. For example, although one model succeeded in reproducing herding using reinforcement learning combined with adaptive neural networks [28], it involved only one dog and one sheep. Another model in our previous work succeeded in reproducing the phenomenological model but at a cost. To increase the probability of success, we introduced a rather explicit switching mechanism as an intermediate target called a subgoal [29].

To construct a new learning model, we focus on the fact that the task of a shepherd dog is assigned by a human, who can train the dog or at least show demonstrations by another dog [30]. Dogs are special because they have coevolved with humans and have an uncanny ability to read socialisation cues and exhibit communicative behaviour with humans [31]. Dogs have worked with humans not only in shepherding but also in other activities such as rescue, sledding, hunting, and guiding [16, 32]. Thus, a dog may be able to learn the policy of switching strategies from humans or possibly from other dogs through demonstrations [16–18, 33].

The task of the shepherd dog is to acquire the strategies shown in the phenomenological model. In this section, we try to determine if it is possible to model the herding behaviour of a shepherd dog given expert demonstrations by proposing an imitation learning model for herding.

3.2 Imitation Learning

Similar to reinforcement learning, imitation learning is formulated assuming a Markov decision process [34]. Let $s_t \in \mathcal{S}$ be the state at time t. An agent selects an action a_t from a set of possible actions $\mathcal{A}$ in accordance with the policy $\pi : \mathcal{S} \to \mathcal{A}$ and the state changes to s_{t+1} at the next time $t + 1$ owing to the action a_t.

The task of an agent is to learn the optimal policy. Although an agent of reinforcement learning is given a reward, an agent of imitation learning is given a series of demonstrations $\{\xi_1, \xi_2, \cdots\}$, where $\xi_t = (s_t, a_t)$ is a state–action pair of an expert. There are various methods of imitation learning.

As an example, the agent can employ behavioural cloning, one of the main methods of imitation learning [22]. In behavioural cloning, an agent has a parametric function as the policy $\hat{\pi}_\theta : \mathcal{S} \to \mathcal{A}$ and updates the parameter θ so that $\hat{\pi}_\theta$ is consistent with the given demonstrations. The update is carried out by an optimisation algorithm such as stochastic gradient descent, which minimises a loss function that expresses the difference between the actions of the agent and the expert for the same state.

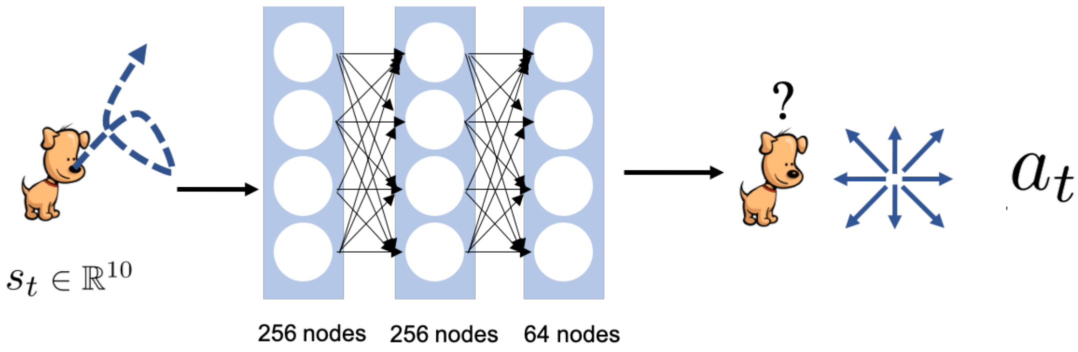

Figure 3.1: Architecture of the map representing policies.

3.2.1 Model

Our dog model is an application of behavioural cloning. We assume a single herder, as in the phenomenological model. The state s_t at time t consists of ten components: the two-dimensional positions of the COM, furthest sheep, final goal, and the dog, and the distance from the COM to the furthest sheep and the distance from the COM to the final goal. The action a_t is one of the eight directions (the cardinal and diagonal directions), where the speed of the dog is 1.5 times that of the sheep.

The map of the dog from a state to an action is represented using a neural network with three hidden layers (256, 256 and 64 nodes) and the rectified linear activation function (Fig. 3.1). The parameters of the neural network are updated so that a loss function is minimised using a stochastic gradient descent method called ADAM [35]. Here, the loss function is the Kullback–Leibler (KL) divergence,

$$\mathcal{L} = \mathrm{KL}(a_t, \hat{a}_t) = \sum_i a_t(i) \log \frac{a_t(i)}{\hat{a}_t(i)}, \tag{3.1}$$

which takes its minimum value if and only if $\hat{a}_t(i) = a_t(i)$, where (i) expresses the eight directions.

3.3 Experiments

In one run, multiple expert demonstrations were generated from the phenomenological model as state–action pairs, where the number of sheep was $N = 25$ and the number of runs was 125. The initial positions of the dog and sheep were randomly chosen from a uniform distribution in the field.

The neural network for the map was trained so that the loss function $\mathcal{L}$ of the observations was minimised. Here, the parameters were updated using ADAM with a learning rate of 10^{-3} in a Python environment with the OpenAI Gym interface.

3.4 Results

In our imitation learning model, we define a task to be successful if herding is completed within 500 steps. The task is unsuccessful, otherwise. In the following analysis, we define the error rates to be the percentage of success subtracted from unity.

The error rates of the shepherding task, averaged over 125 runs, decreased with increasing number of demonstrations t (Fig. 3.2a). The shepherd dog successfully reproduced the phenomenological policy by imitation learning with a learning curve of order t^{-2} (Fig. 3.2b). In addition, the variance of the error rate also decreased (Fig. 3.2c), showing proof of stable learning.

To examine how the dog learnt the switching policy, details of the emergent behaviours are shown in terms of the distances from the COM to the furthest sheep and final goal at different times during the training process (Fig. 5).

In the early stage with a few demonstrations (typically less than 30), the distance from the COM to the furthest sheep increased monotonically throughout the trajectory (Fig. 5a). This means that the shepherd dog did not learn the collecting strategy. We further observe that the distance from the COM to the final goal increased which means that the herd moved further away from the goal (Fig. 5a). This also means that the driving strategy was not learnt. Sample trajectories of both the phenomenological and proposed models show a large variance despite starting from the same position (Fig. 5b).

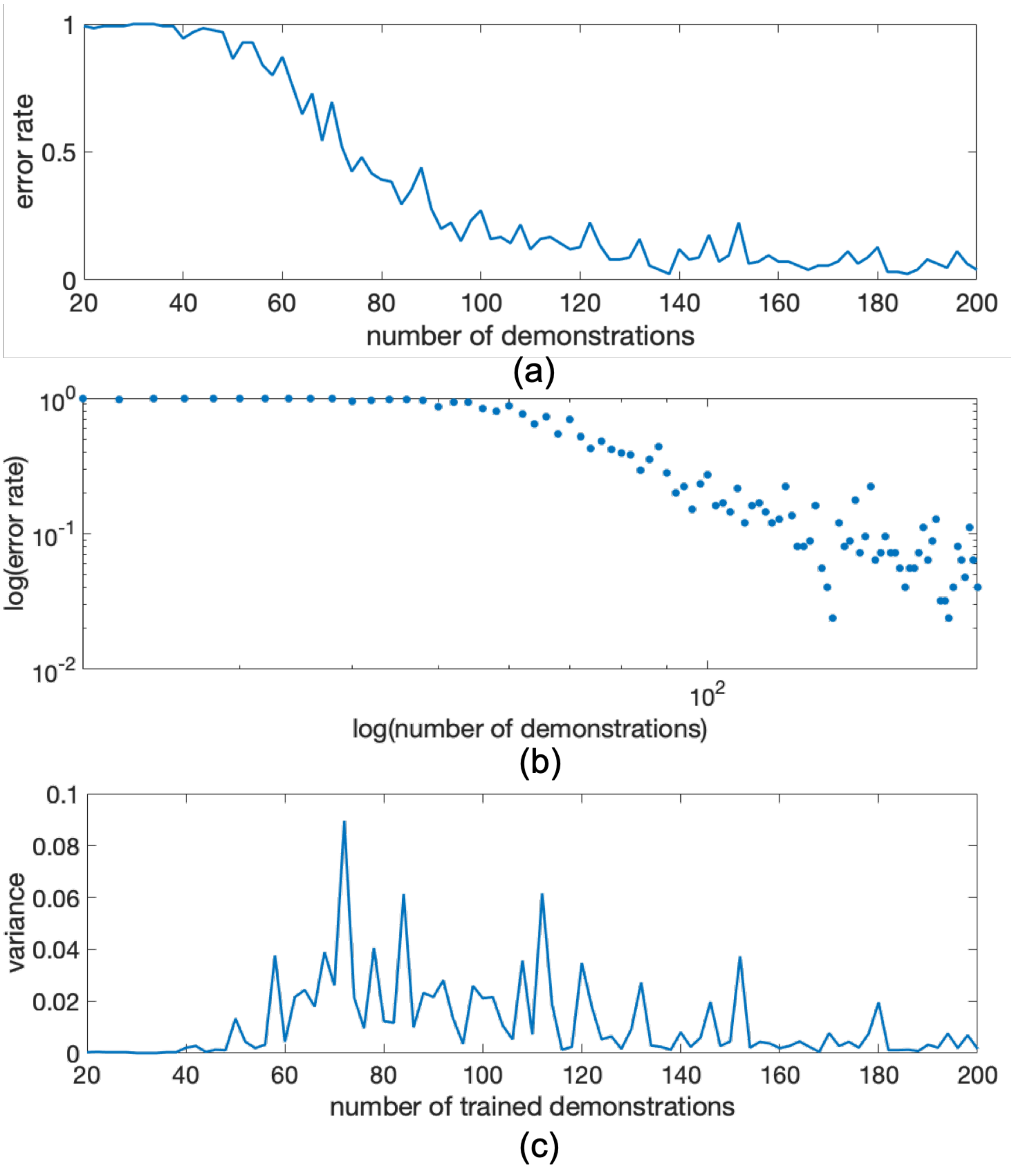

Figure 3.2: (a) Error rate as a function of number of demonstrations. (b) Log-log plot of the error versus the number of demonstrations. (c) Variance of the error as a function of the number of demonstrations.

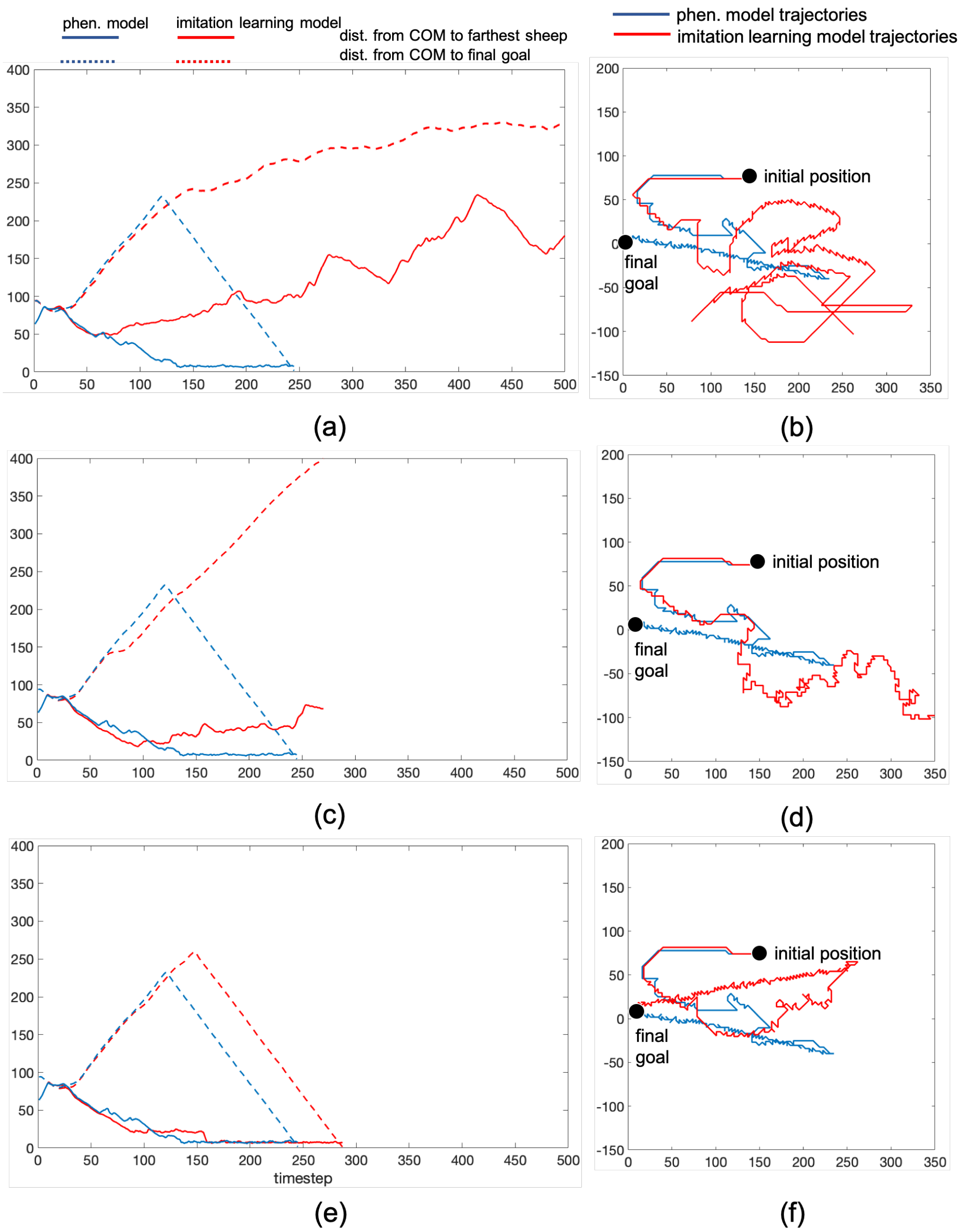

Figure 3.3: (a)(c)(e) Distances from the COM to the furthest sheep and final goal and (b)(d)(f) typical trajectories of the dog with the phenomenological model and our proposed model for (a)(b) 20 demonstrations, (c)(d) 100 demonstrations and (e)(f) 150 demonstrations.

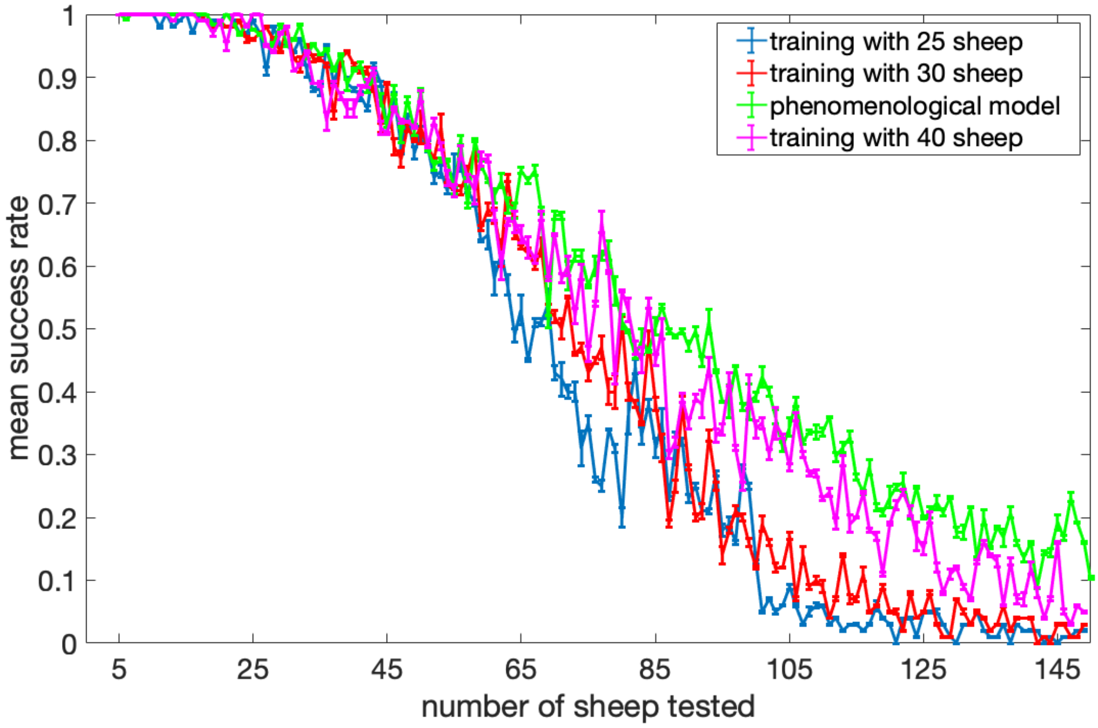

Figure 3.4: Success rates of the proposed model trained under different conditions.

In the middle stage with around 100 demonstrations, the distance from the COM to the furthest sheep decreased and then became almost constant (Fig. 5c). This means that the distribution of the sheep was kept constant and that the shepherd dog successfully performed the collecting strategy. However, it was stuck at this stage and unable to subsequently drive the flock to the goal. The shepherd dog eventually brought the herd further away from the final goal (Fig. 5d).

In the final stage, the distance from the COM to the final goal first increased during the collecting process and then decreased as the shepherd dog drove the herd to the final goal (Fig. 5e). The trajectories of both the phenomenological and proposed models varied despite having the same initial position (Fig. 5f), possibly due to the stochasticity of the movements of the sheep. This implies that the dog did not simply copy the demonstrations but learned what it should do.

3.4.1 Generalisability

To confirm our hypothesis that the dog learned the required task, we carried out more experiments, where the shepherd dog was given 25, 30 and 40 sheep in the training phase and attempted to herd 5 to 150 sheep in the test phase. It was found that the dog trained with 25 sheep successfully herded up to 70 sheep 50% of the time and up to 100 sheep 20% of the time (Fig. 6). Although the success rates decreased as the number of sheep increased, the rates were comparable to those of the phenomenological model, implying that the dog did not simply copy the demonstrations but learned the required task.

3.4.2 Limiting Conditions

Since our expert demonstrations are based on phenomenological models with $N = 25$ sheep, we examine some limiting conditions to verify the performance of our model.

First, we consider the case when the expert demonstrations have $N = 25$ sheep, our neural network trains only a minimum of three sheep. We examine how our model will perform when tested from 3 to 85 sheep as shown Fig. 3.5.

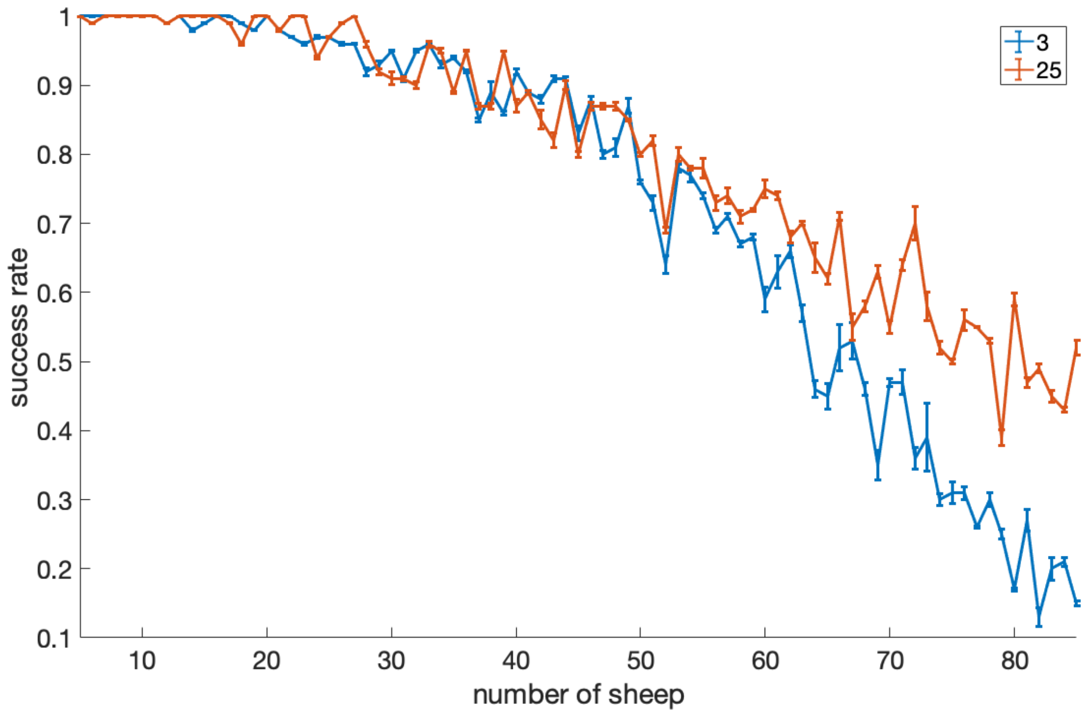

Figure 3.5: Performance of the model when observing 25 sheep from the phenomenological model, but only training N sheep.

We see the results are expected: that by training only a very few number of sheep, performance is inferior compared to training more sheep.

Second, we consider the other extreme case when the expert demonstrations provided only have N sheep, and we train with only N sheep. Our results show that success is very limited. By observing and training only 3 sheep, learning is extremely unstable and herding cannot be performed successfully in most cases beyond 5 sheep. By observing and training more sheep from the phenomenological model, performance vastly improves. This can be attributed to the number of states visited by the dog. In the case of observing only three sheep, the number of states visited and observed by the dog is extremely small and is unlikely to generalise the strategies learnt when more complicated configurations are introduced as a result of increasing the number of sheep. As we can see in the 3.4, increasing the number of sheep can only contribute to the success marginally. One reason for this is that no new further states are observed by the dog, and

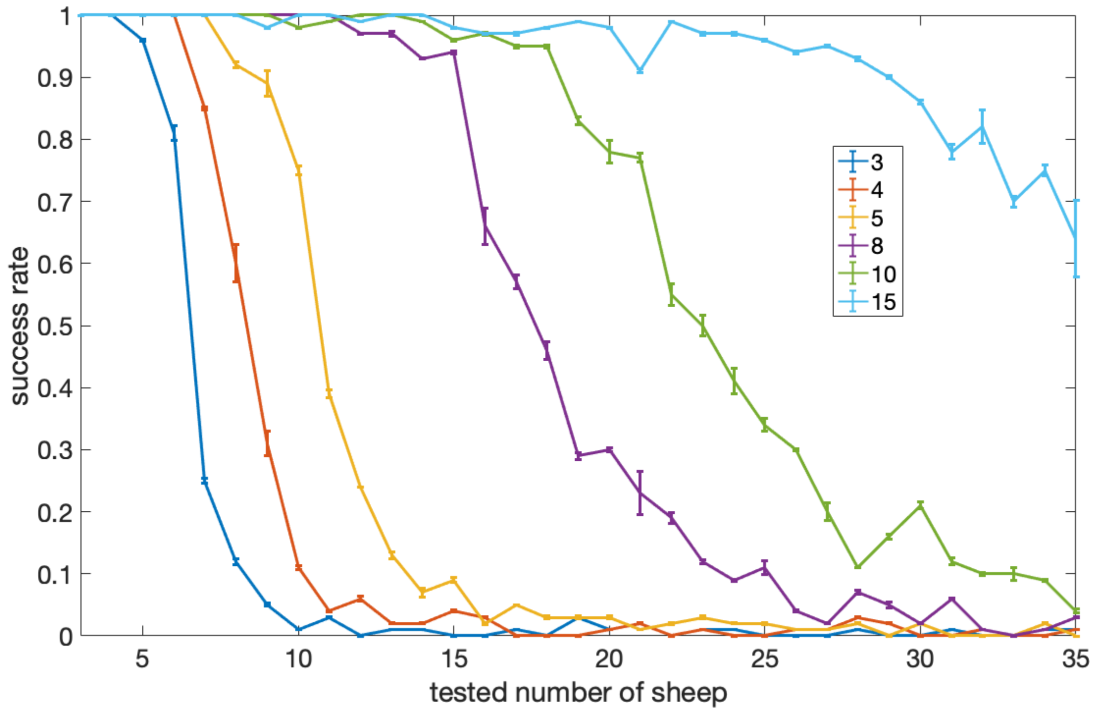

Figure 3.6: Performance of the model when observing N sheep from the phenomenological model and only training N sheep.

that all information has been known already.

3.5 Discussion

In this section, we proposed an imitation learning model that learns the policy for the herding task based on demonstrations by experts. In our model, the policy, which is a map from a state to an action, was implemented by a neural network and its parameters were updated by stochastic gradient descent. Through our experiments, we show that after training the learning model, the dog was able to reproduce the policy of switching between collecting and driving according to the observed state of the sheep. In addition, we are able to show that the dog did not simply copy the demonstrations but learnt the policy by generalising the learning model to herd more sheep than what was shown in the demonstrations.

In this imitation learning model, we assumed a single herder performing the herding task. Our results were able to show that the herder is able to perform the herding successfully with a rate of 50% when herding some 80 sheep. In reality, more herders may be needed when more sheep are needed to be herded efficiently. As a result of this increase, new strategies may be necessary, which may not be observed from the phenomenological model itself.

3.6 Conclusion

In this chapter, we created an imitation learning model of the herding of sheep by a shepherd dog. Given a set of expert demonstrations based on the phenomenological model of herding, we trained a neural network to successfully recreate the strategies needed to complete the herding task. The performance of our model shows a logistic increase as the number of demonstrations provided by the expert also increases. In addition, variance of the error decreases over time indicating a more stable learning. Furthermore, our learning model is able to generalise the herding task. Despite being shown only 25 sheep as expert demonstrations, our imitation learning model is able to herd more sheep than this and approximate its performance to the phenomenological model.

We also identified some emergent behaviours which add possible insights into the model. The shepherd dog first learns to collect the flock of sheep and prevent it from dissipating. It then eventually learns to perform both the collect and drive strategies to complete the task.

4 Herding Behaviour in a Horse Harem

4.1 Introduction

In this chapter, we focus on the followers. They are members of animal groups that respond to the initiation of motion by the herder. Many species across the animal kingdom exhibit different forms of collective motions under various circumstances. Some maintain a cohesive group through subtle interactions among conspecifics [36, 37] and others perform group aggregation and evasion in the face of a threat to lower the risk of predation [5, 6, 19].

To examine their responses to various situations, animal groups have been tracked remotely with high spatial and temporal resolutions [38] using advanced technologies such as GPS technology [37, 39, 40], imaging algorithms [41–43] and unmanned aerial vehicles [44, 45]. GPS devices attached to animals, for example, have uncovered leader–follower relationships within the group during motion, that is, certain individuals within a group of pigeons contribute more to decision-making during their flight, while others consistently copy movements [37]. In another example, roles in a group of pet dogs may change while walking, but relationships among conspecifics are stable in the long term [39].

Recent advances in technology have allowed us to track and analyse the motion of animals, endangered or otherwise, in restricted spaces or in the wild [45]. Coupled with tracking software [43, 46–48], this relatively new method offers a noninvasive way to track and analyse animal groups discreetly in their natural habitat [44].

Since ground observations may have a potential source of error [49], aerial observations using a unmanned aerial vehicles, or drones, are preferable for obtaining an unbiased view of absolute movements of the animals in question. In addition, horses live in vast open areas, which makes it possible to locate all their absolute positions from the air.

During herding, different forces of interaction come into play, reflecting the dynamics of the movements of the individual animals. In the case of horses, a stallion chases the mares to herd them [13]. Herding among horses is thought to be performed primarily by the stallion [12, 14, 15] to protect the flock from other herds [3, 50], for group cohesiveness with the aim of long-term social relationships with members of the group [12], or to drive the herd to another location for various reasons. Feral horses naturally acquire the instinct to herd, unlike shepherd dogs in the previous chapter.

Several anecdotal accounts of the herding of horses are available. Many studies are qualitative in nature and describe the physical attributes of stallion behaviour while herding mares [51]. These reports include descriptions of the position of the ears or the general posture of the horse while performing tasks [12,51]. In addition to these qualitative aspects of herding, a quantitative treatment is preferable to obtain an overall understanding of the behaviour.

In this chapter, we analyse the movements of female feral horses (*Equus caballus*) of the Garrano breed during a herding event initiated by a stallion using collected videos. We propose a mathematical model of the herding behaviour of Garrano horses to quantitatively explain how mares move in the presence of a herding stallion. The model is based on a heuristic model of herding of sheep by a shepherd dog [19], that is, the motion is a linear combination of various forces, which is modified to accommodate differences in the motion between sheep and mares.

4.2 Materials and methods

4.2.1 Subjects

Incidents of horse herding were obtained by filming from a drone (Phantom 3 Advanced and Mavic Pro, DJI China) in Serra D'Arga, Portugal (8°42′N, 41°48′E). The horse habitat includes grass fields, shrubs, forest and shrub areas, where about 200 horses live. Our focal group was a harem group comprising one adult male, five adult females and one foal filmed in 2016, and one male, six adult females, one sub-adult female and two foals in 2017.

Data were taken during the breeding season (June to July) in 2016 and 2017. The focal group was recorded every 30 min using a drone, which took off approximately 10–30 m from the horses and flew at a height of about 75 m for 10–15 min until the battery was depleted. All individual horses were identified as well as the time points when herding began and ended. Observations lasted 6–9 h a day, and 3–10 flights of the drone were required per day for 30 days. A total of 59 h of video footage was recorded.

All instances of herding were manually identified from the high-resolution video. Herding began when the stallion lowered its head with its ears pinned to the back [12,51] and ended when its head returned its normal position. All movements of female horses were tracked and analysed only during instances of herding by the stallion.

Only moving adult mares were included in this study. Individuals that did not move more than 2 body length during a herding instance were not considered in this study.

4.2.2 Image processing

A total of 12 instances of herding were recorded among the video clips. To reduce the effects of drone movements such as rotation and tilting due to wind and other factors, the videos were preprocessed for stabilisation using the Warp Stabilizer function in Adobe Premiere Pro. The resulting stabilised videos had a frame rate of 30 frames per second.

After the preprocessing, each of the horses in the video was tracked frame by frame using Tracker [46], a free video-analysis software built on the Java framework. Two additional stationary objects on the ground (rocks) were also tracked to ensure no further rotation or tilting remained. Any such drone movements were mathematically corrected using rotation matrices. Tracker then gave the x and y coordinates of individual horses, where the scales of videos with different heights were normalised with the male's body length in each video as the standard unit.

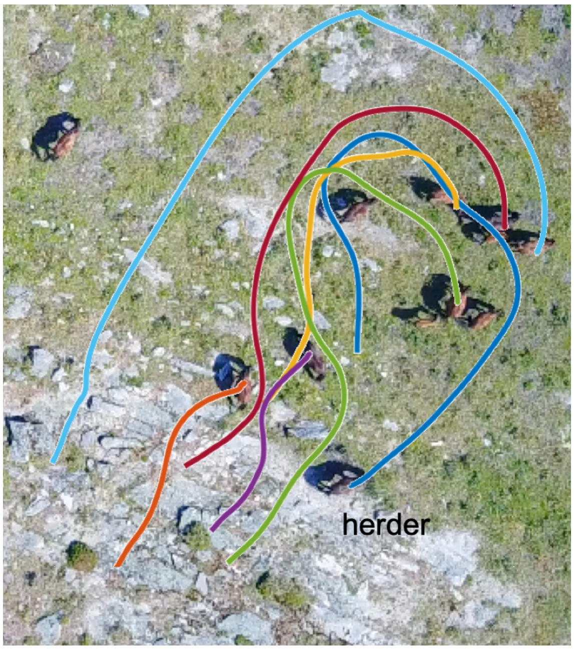

Figure 4.1: Extracted trajectories of horses during a herding instance superimposed on a drone video screenshot.

Finally, the tracked positions of the individual horses were postprocessed using MATLAB (release R2017b). To smoothen the data and reduce the effects of noise from the measurements, data was filtered using a Butterworth filter with order 2 and a sampling frequency of 30 Hz (Fig. 4.1). The filtered data were used to calculate various characteristics of horse movements such as the speed and orientation.

4.3 Model

Our proposed model for the motion of mares while being herded by a stallion is based on the phenomenological model of herding by a shepherd dog [19]. The latter model assumes that individuals being herded move with a constant velocity as seen in (Fig. 4.2). We take note, however, that the horses move with a variable speed. Hence our proposed model reflects these variations in position.

Our model for female feral horses during herding by a stallion is based on the

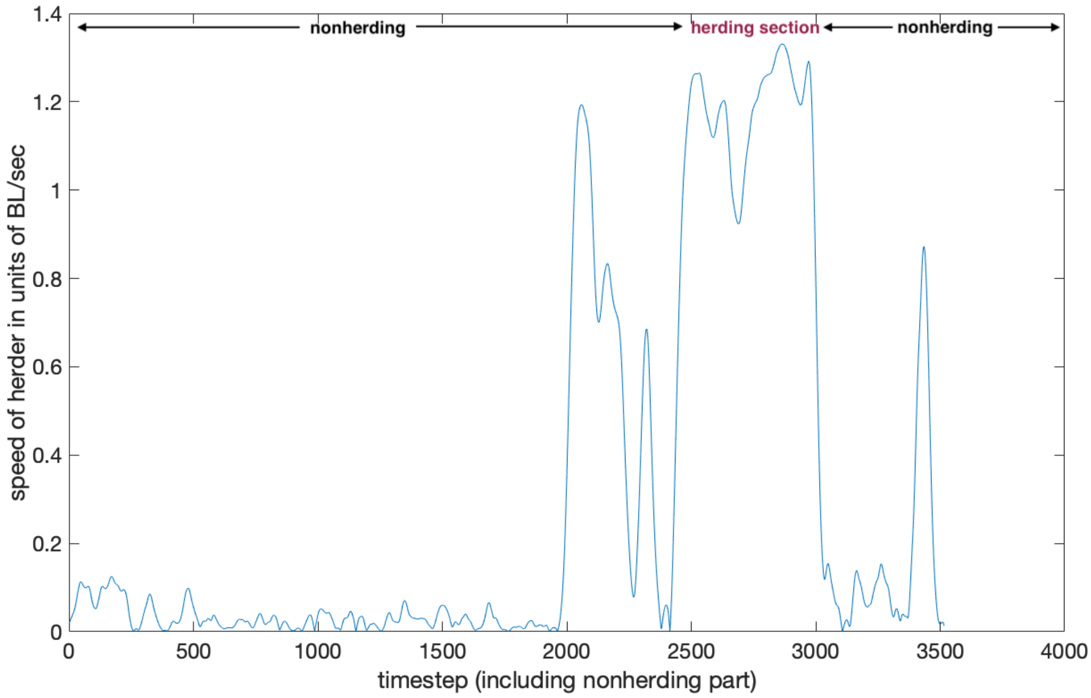

Figure 4.2: An example of variable speed of a female horse during while being herded by a stallion.

model for sheep during shepherding by a shepherd dog where the sheep model assumes a constant speed and the direction of motion is determined by a linear combination of the following components: inertia, a repulsive force from the shepherd dog, a short-range repulsive force, an attractive force to the centre of the group (centre of mass, COM) and a random noise.

Let each mare be M_i, the stallion H. We enumerate the forces of interaction as follows:

1. An inertial force which attracts each mare to its previous heading.

2. When a female horse is within a certain distance d_{rep} from the stallion, it experiences a force of repulsion, $R^h_{i,\text{rep}}$, directed away from the stallion,

$$R^H_{i,\text{rep}} = M_i - H. \tag{4.1}$$

3. A female horse experiences a force of repulsion, $R^M_{i,\text{rep}}$ from other female

horses when they are within the repulsion zone,

$$R^M_{i,\mathrm{rep}} = \frac{1}{n_{\mathrm{rep}}} \sum_{j=1}^{n_{\mathrm{rep}}} \frac{M_i - M_j}{|M_i - M_j|}, \tag{4.2}$$

where n_{rep} is the number of female horses within the zone, as seen in Fig. 4.3.

4. A female horse experiences a force of attraction, $A_{i,\mathrm{att}}$ from other female horses within the attraction zone,

$$A_{i,\mathrm{att}} = \frac{1}{n_{\mathrm{att}}} \sum_{j=1}^{n_{\mathrm{att}}} \frac{M_j - M_i}{|M_j - M_i|}, \tag{4.3}$$

where n_{att} is the number of female horses within the zone, as seen.

5. We introduce the synchronisation attraction, where each horse matches its direction with the nearest moving conspecific within a set region Fig. 4.3.

6. Finally, each mare experiences a force of attraction towards the group's centre of mass, COM,

$$C_i = \mathrm{COM} - M_i. \tag{4.4}$$

Note that the random noise component was omitted since the trajectories of horses were sufficiently smooth. In addition, all the components were normalised to allow our coefficients to be compared reasonably.

4.3.1 Parameter estimation

Our proposed model is a linear combination of the enumerated forces above, $F^{(T)}_{xi}, F^{(T)}_{yi}$, in two dimensions x, y, at each time step $T = 1, 2, \cdots, t$, where $i = 1, 2, \cdots, 6$.

$$c_1 \left[F^{(T)}_{x1}, F^{(T)}_{y1}\right] + \cdots + c_6 \left[F^{(T)}_{x6}, F^{(T)}_{y6}\right] = \left[v^{(T)}_{x1}, v^{(T)}_{y1}\right], \tag{4.5}$$

or

$$\begin{bmatrix} F^{(1)}_{x1}, F^{(1)}_{x2}, \cdots, F^{(1)}_{x6} \\ F^{(1)}_{y1}, F^{(1)}_{y2}, \cdots, F^{(1)}_{y6} \\ \vdots, \ddots \\ F^{(t)}_{x1}, F^{(t)}_{x2}, \cdots, F^{(t)}_{x6} \\ F^{(t)}_{y1}, F^{(t)}_{y2}, \cdots, F^{(t)}_{y6} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_6 \end{bmatrix} = \begin{bmatrix} v^{(1)}_{x1} \\ v^{(1)}_{y1} \\ \vdots \\ v^{(t)}_{x6} \\ v^{(t)}_{y6} \end{bmatrix}, \tag{4.6}$$

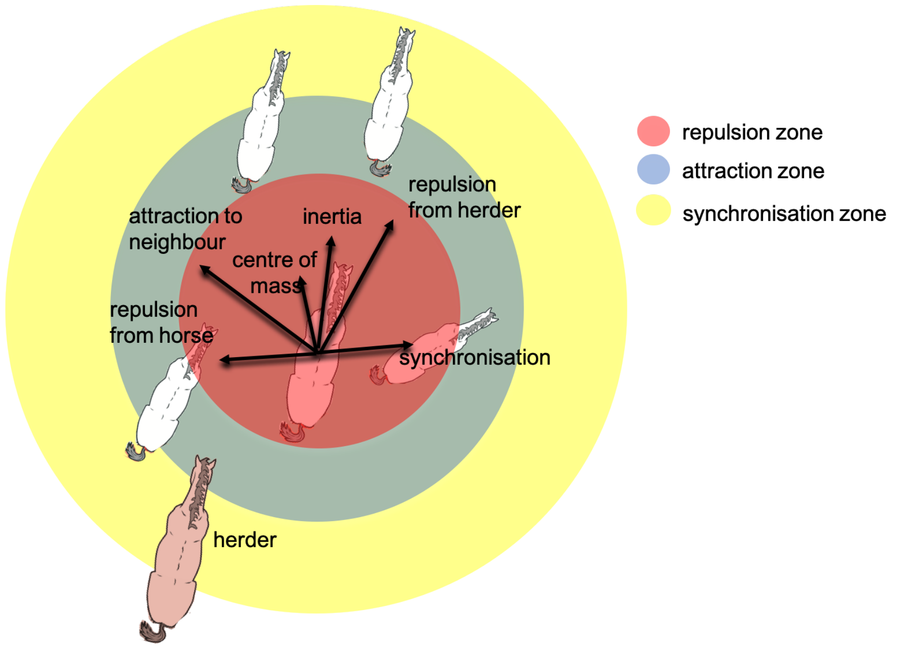

Figure 4.3: Schematic diagram of the zones and forces of interaction experienced by a mare while being herded by a stallion.

or simply $\mathbf{Fc} = \mathbf{v}$, where $\mathbf{F}$ comprises the components of the interaction, $\mathbf{c}$ comprises the parameters expressing the coefficients of the linear combination and $\mathbf{v}$ comprises the velocities of the individuals.

The coefficients should be estimated from the observed data. To do this, we minimised the error between the model and the measurements under the condition that the coefficients are non-negative, that is,

$$\underset{\mathbf{c}}{\operatorname{argmin}} ||\mathbf{Fc} - \mathbf{v}||_2, \quad \text{subject to} \quad \mathbf{c} \geq \mathbf{0}. \tag{4.7}$$

Note that only moving horses were considered in the estimation of the coefficients.

The coefficients of the components were estimated using the data and the trajectories reproduced by the model were compared with the observed trajectories. For the estimation of coefficients, two cases were considered: in the first case, each horse in each herding instance has a specific coefficient set; in the second case, each horse has the same coefficient set in all herding instances, which is called the shared coefficients in the following.

The other parameters of our model were manually identified from the data sets as follows: the radius of repulsion (4 BL), the radius of attraction (7 BL), the herder detection distance (9 BL), and the region of synchronisation (10 BL).

4.3.2 Evaluation

The trajectories reproduced using the model were evaluated in terms of the root-mean-square error,

$$\text{RMSE} = \sqrt{\sum_t \left(\hat{P}_i(t) - P_i(t)\right)^2}, \tag{4.8}$$

where $\hat{P}_i(t)$ is the predicted position of female i at time t and t is the herding duration.

4.4 Results

4.4.1 Properties of the data

By applying the image processing techniques outlined in Section 3.1.2 to our data sets, we obtained 12 sequences of herding. Initial distributions of the mares

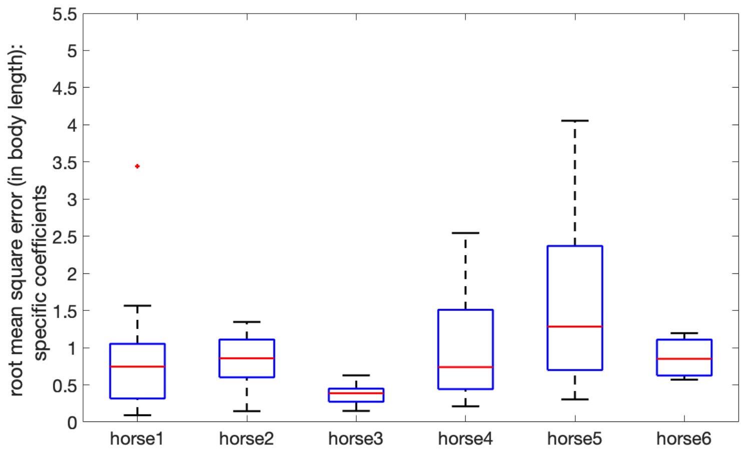

Figure 4.4: Distribution of the RMSE for each horse using specific coefficients.

vary widely, with the most compact initial distribution having a mean pairwise distance of 3.671 body lengths (BL) and a standard deviation (SD) of 1.175 BL. Meanwhile, the most dispersed initial distribution had a mean pairwise distance of 31.779 BL with an SD of 13.075 BL. The herding instances ranged from 4.67 to 23.3 s, with an average of 14.48 s and with an SD of 6.82 s.

4.4.2 Model with specific coefficients

In this model, we calculated the coefficients *specific* to each horse during each herding instance. Most of the obtained root mean square error (RMSE) for this model were within 2 BL (Fig. 4.4). These RMSEs are sufficiently small to reproduce the observed data (Fig. 4.5). The average RMSE in this model was 0.90 BL with an SD of 0.81 BL.

4.4.3 Model with shared coefficients

Here, we calculated the coefficients for each horse *shared* across all herding instances. We summarised the obtained coefficients in Table 4.1.

The RMSEs of the model obtained from the observed data (Fig. 4.6) has an average RMSE of 1.68 BL and an SD of 1.04 BL. Although the RMSEs were

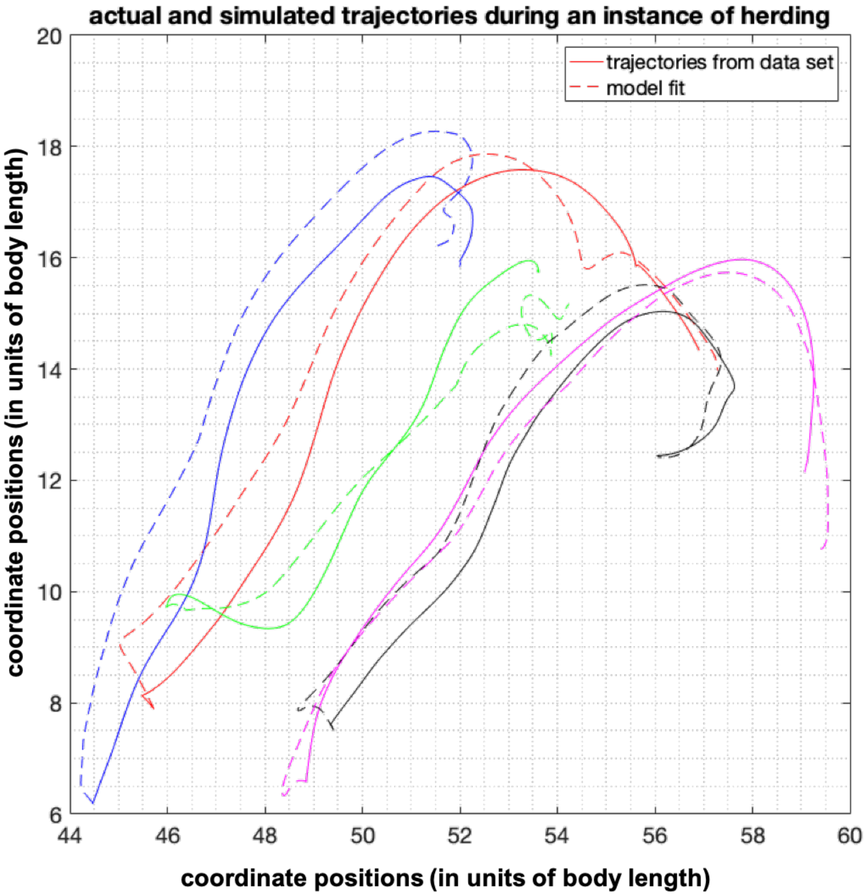

Figure 4.5: Comparison of actual horse trajectories with the best-fit model using specific coefficients.

Horse identity	Inertia	Short range repulsion	Medium range attraction	Repulsion from stallion	Attraction to COM	Synch. attraction
1	0.600	0	0	0.009	0.051	0.134
2	0.499	0.033	0	0.183	0.036	0.083
3	0.353	0.204	0.024	0.399	0.592	0.015
4	0.595	0.230	0	0.359	0.423	0.125
5	0.912	0.037	0	0.399	0.287	0
6	0.677	0	0.027	0.272	0.298	0

Table 4.1: Coefficients of the forces obtained using six-component model with shared coefficients.

larger than those of the first model, their trajectories were of similar trends to the observed data as seen in (Fig. 4.7).

To compare the similarity between predicted and actual trajectories, we calculated the directional error at each time t, defined as

$$E = 1 - \frac{\mathbf{v}_m \cdot \mathbf{v}_d}{|\mathbf{v}_m||\mathbf{v}_d|}, \tag{4.9}$$

where $\mathbf{v}_m$ and $\mathbf{v}_d$ are the velocity vectors of each horse in the model and actual trajectories, respectively. As a result, the directional errors of the horses were sufficiently small almost everywhere (Fig. 4.8) and the average error over all moving horses across all herding instances was 0.031. This means that the predicted trajectories run in the same direction as the actual trajectories.

4.5 Discussion

In this chapter, we proposed two mathematical models to explain the motion of mares as a response to a herding stallion. The models are a linear combination of six components based on the previous models in [8, 19, 42] and modified so that the velocity is not constant. The coefficients of the six components were calculated by non-negative least-squares optimisation from the observed data in two ways. The coefficients in the first model were calculated specific to each

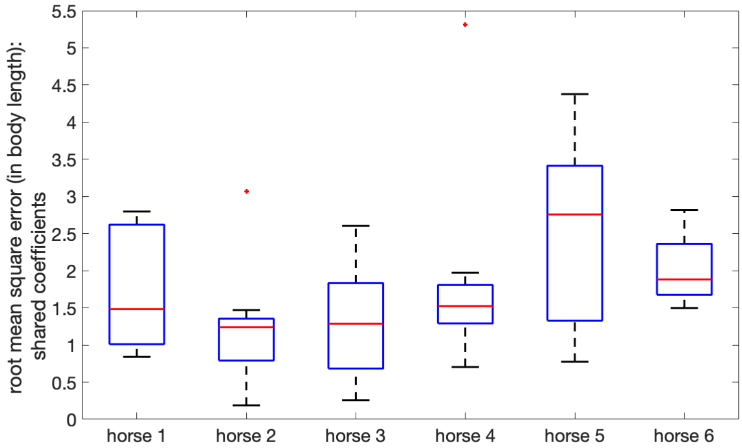

Figure 4.6: Distribution of the RMSE for each horse using shared coefficients for one horse.

individual in each herding instance, and those in the second model were calculated for each individual shared across all herding instances. As a result, the model with specific coefficients reproduced the observed trajectories more closely than the shared coefficients model.

We introduced two ways of estimating the coefficients, specific coefficients and shared coefficients. To determine which is better, we examined their distribution by visualisation using principal component analysis (PCA). We found that the coefficient vectors of the horses have a large variation, implying that the model overfits the observed data (Fig. 4.9).

To examine the personalities of females, we compared their coefficient vectors (Table 1), which appeared to be widely distributed. This wide distribution was confirmed by the hierarchical clustering of the coefficient vectors plotted in a dendrogram (Fig. 4.10). Although two females have similar coefficient vectors, the other four females have very different coefficient vectors, indicating their individualistic personalities.

Our model consists of six components based on the previous models in [8,19,42], which is consistent with the previous findings [13]. However, the medium-range attraction took values of zero for four mares and less than 0.03 for the other two mares. Since each component is normalised, this value expresses the contribution

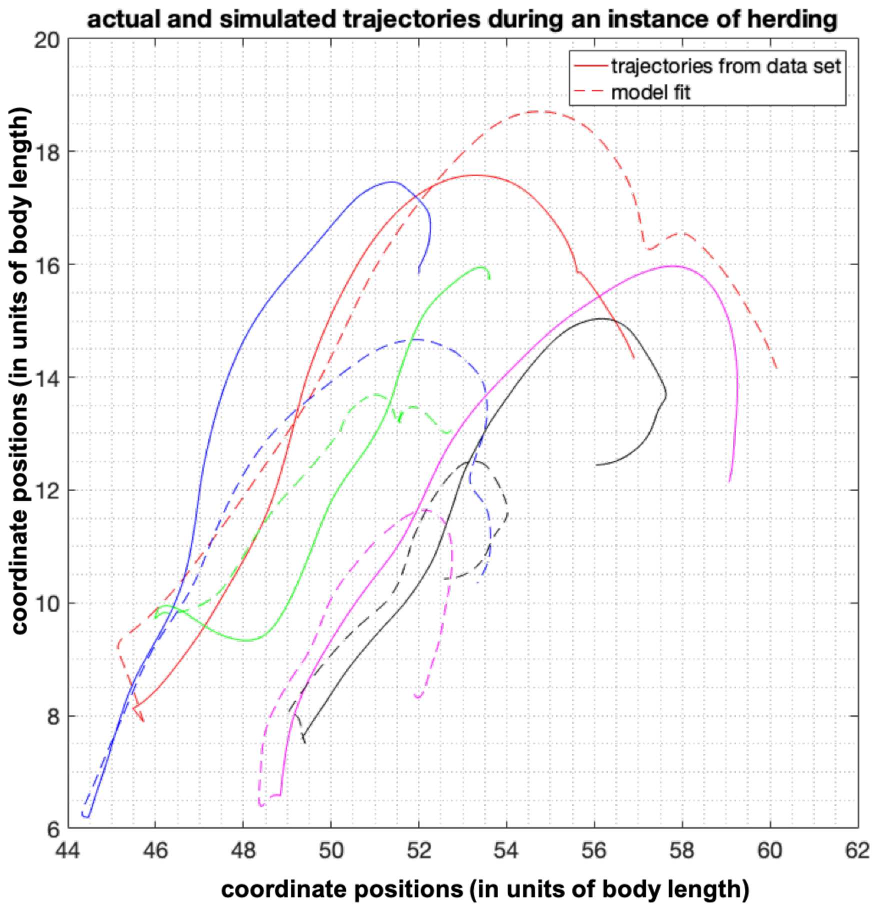

Figure 4.7: Comparison of actual horse trajectories with the model using shared coefficients for each horse across herding instances.

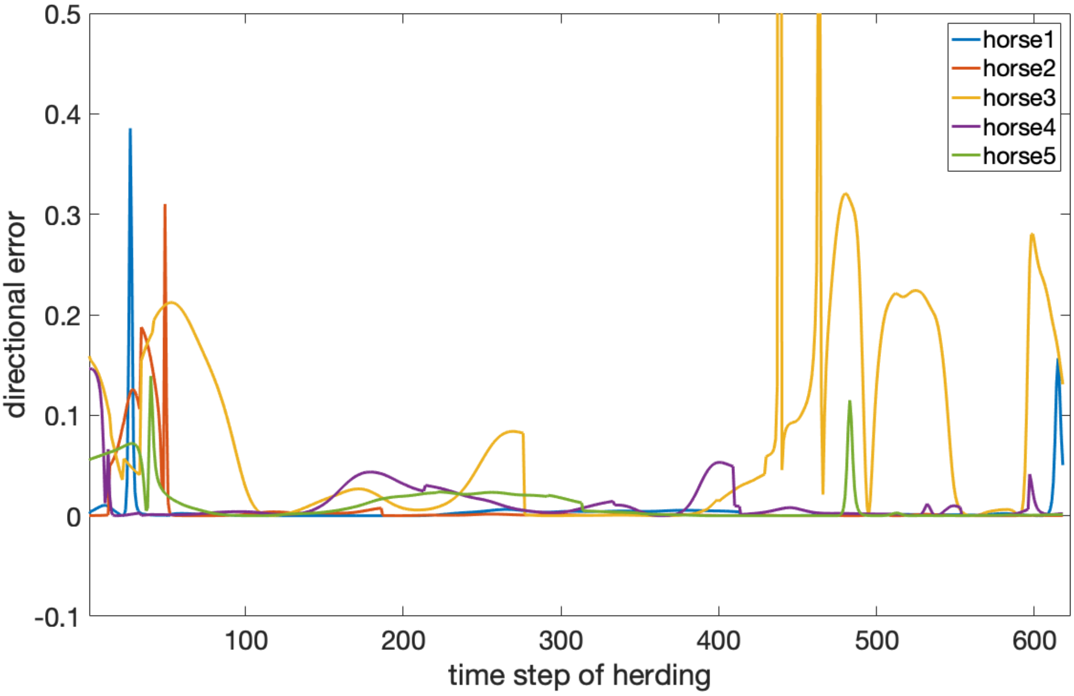

Figure 4.8: Directional error per horse in one herding instance.

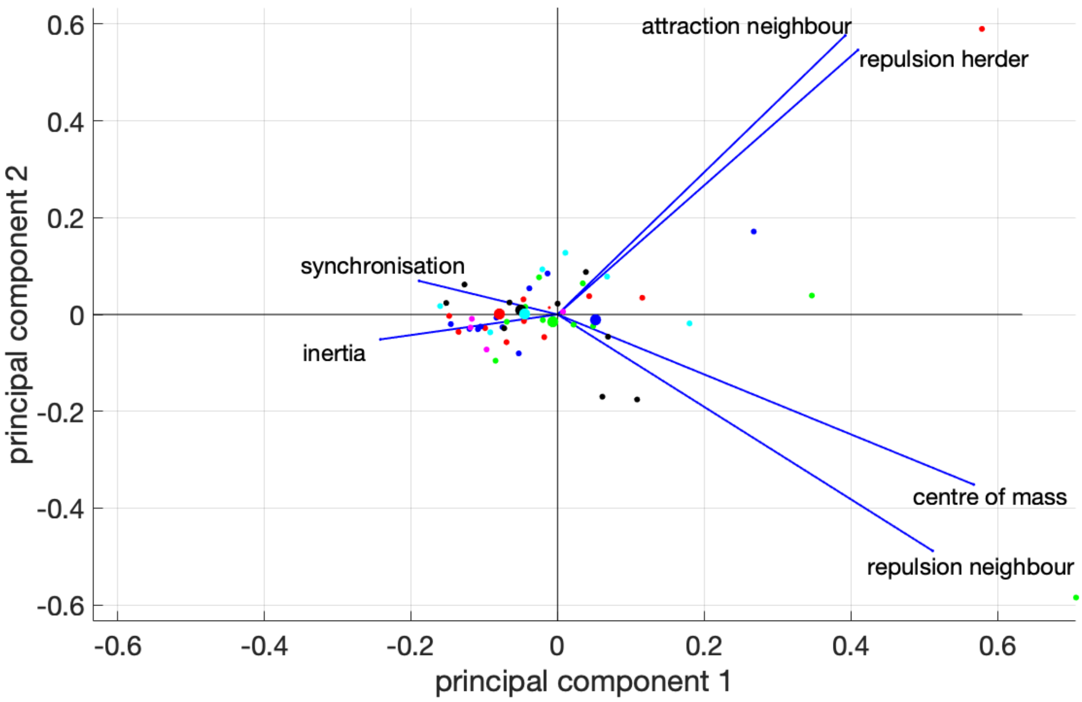

Figure 4.9: Distribution of estimated coefficient vectors after PCA. Small circles show different coefficients and large circles show common coefficients. Each color indicates a different individual.

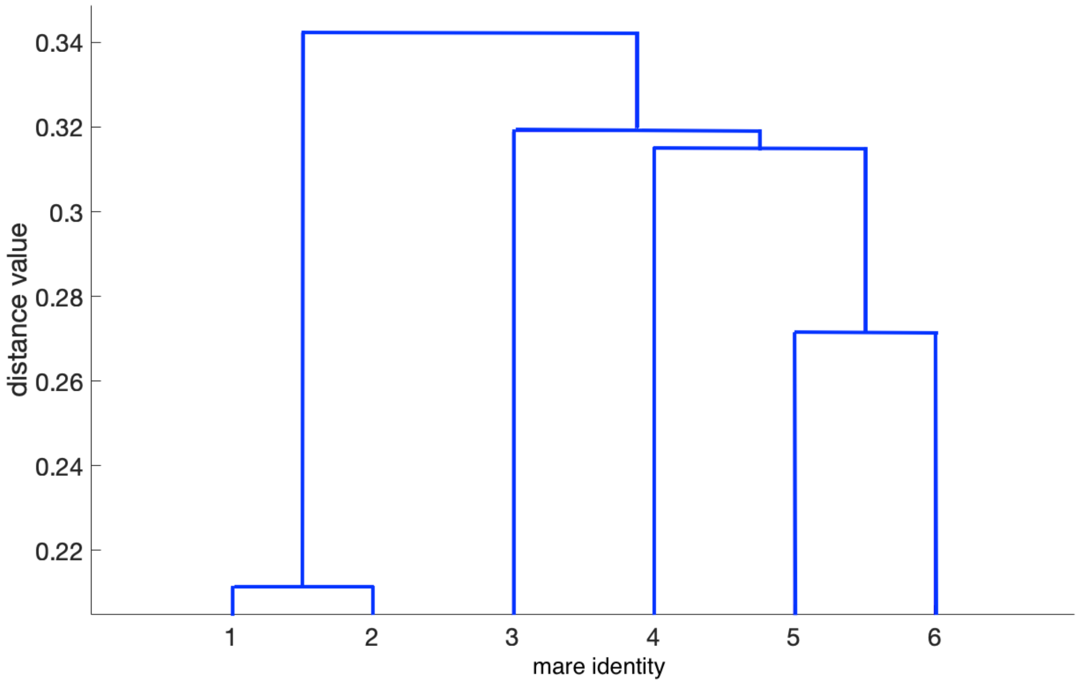

Figure 4.10: Hierarchical clustering of horses based on the coefficients of the second model

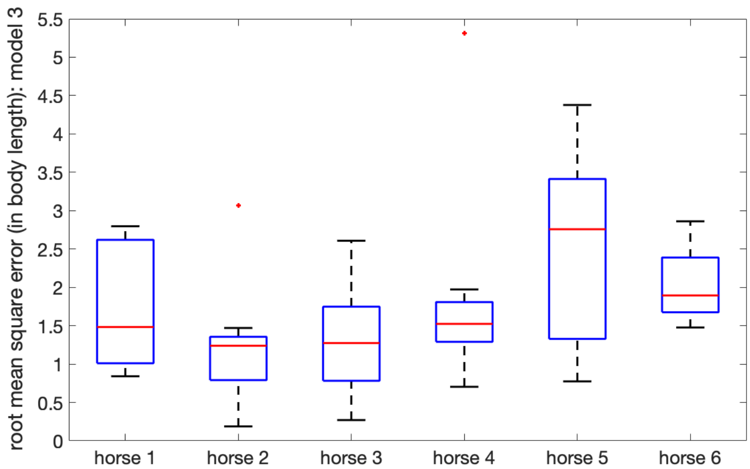

Figure 4.11: Distribution of errors when the attraction term is removed.

Horse identity	Inertia	Short-range repulsion	Repulsion from stallion	Attraction to COM	Synchronisation attraction
1	0.600	0	0.009	0.051	0.134
2	0.499	0.033	0.183	0.036	0.083
3	0.354	0.210	0.402	0.620	0.014
4	0.595	0.230	0.359	0.423	0.125
5	0.912	0.037	0.399	0.287	0
6	0.677	0	0.281	0.330	0

Table 4.2: Coefficients of the forces obtained using five-component model with shared coefficients.

to the movement. This suggests that the medium-range attraction is redundant, in other words, it can be removed from the model. To confirm this, we modified our model by removing this attraction and calculated the RMSE of the simplified model (Fig. 4.11). As a result, the RMSE was still 1.68 BL with an SD of 1.03 BL. Thus, we can ignore the medium-range attraction. The resulting coefficients of the corresponding forces are presented in Table 4.2.

The novelty of this work lies in treating the horses with non-constant velocities unlike in [19]. Given our data sets, we are able to compute for the contributions of the forces of interaction. They give us an understanding of the different forces in play while they are being herded by the stallion.

In the section on horses, our study is constrained by the difficulty of obtaining data sets in the wild. We have created a Newtonian model to explain the motion of the horses as a result of herding by a stallion. This means that the treatment is microscopic in nature, and modifications maybe be needed when more horses are introduced into the analysis.

When more horses are introduced, a more statistical approach can be taken to include a more macroscopic analysis of the group. With this, modifications will have to be made to include and analyse more forces among subgroups within a harem.

Furthermore, one limitation that our study has is the use of only one time-stationary coefficient to describe the entire herding instance. In reality, we surmise that the coefficients would change more dynamically according to different circumstances and activities of the individual horses. This could be something worth exploring in the future.

4.6 Conclusion

In this chapter, we modified the phenomenological model of herding to be able to describe the motion of mares in a horse harem as a response to the herding of a stallion. We identified six forces of interaction, namely: inertia, repulsion from stallion, repulsion from neighbouring horses, attraction to neighbouring horses, attraction to the centre of mass, and a force of synchronisation. We created a mathematical model to describe the velocity of the mares as a linear combination of the six forces of interaction.

We presented the result in two ways. First, we calculated the coefficients of the forces of interaction for each horse and for each specific herding instance. This gave us the best-fit *specific* coefficients, where most of the actual and model trajectories differ only by at most 2 BL. Although this result gives the least possible error, it is not able to generalise well to other herding data sets.

This leads us to the second result. We calculated the coefficients of the forces by combining all the herding data we have acquired per horse. This means that each horse will have a set of coefficients *shared* across all herding instances. In this case, the root mean square error is greater than that of the previous case, but we are able to generalise the model to include other herding instances.

We have validated our model by computing for the directional error between the actual and model trajectories. Despite the differences in scale between the actual and model trajectories, our model is able to recreate directional trends.

We also conclude that since the coefficients associated with attraction to neighbouring horses is low, we can further simplify our model to include only five forces of interaction.

5 Conclusion

5.1 Summary

We presented two studies on collective behaviour in this dissertation. First, we focused on the herder of a herding task. Here, we created an imitation learning model to possibly explain how a shepherd dog learnt the strategies to complete the task. During the learning process, we observed two emergent behaviours that provides insights into the learning. The herder learnt to collect the group and keep the members from diverging. It then eventually learnt to drive the group towards the final goal position. Despite being shown only 25 sheep as demonstrations, our learning model was able to generalise to herd from 5 to 150 sheep with a performance approximate to that of the phenomenological model.

The second part of this dissertation focused on the followers: mares in a horse harem. We created a mathematical model to explain what influences the motion of mares while being herded by a stallion. Our model is able to recreate trajectories and directional trends of the mares' motions.

5.2 Future Directions

There are several possible next steps to extend this study in the future.

In the horse herding study, we can model the motion of the stallion, too. With enough data, we can identify their strategies of herding, and compare similarities to the dog/sheep study.

One extension is to the case where one herder herds many (typically $N > 200$) sheep. In this case, the policy of dividing the entire flock into clusters and herding them one after another is effective [19]. Another extension is to increase the number of herd dogs, which is practical in real shepherding. However, in cases when an even larger flock of sheep is to be herded, a single shepherd dog may not be able to perform the job. Here, multiple shepherds may be introduced and their interaction rules may be altered to accommodate information on fellow herders. In this case, their interactions such as cooperative and competitive behaviour should also be considered [52–54]. Such interactions will be of particular interest from a game theory perspective, where the interaction among the herders and sheep can be studied.

Another extension is to make the sheep learn. Although the behaviour of the sheep was fixed in this study, they may have learnt it through actual experience. This problem can be formulated in terms of a multiplayer game [55].

Our model assumes a fully observable state space, that is, the position of each sheep is known to the dog. However, this setting is too optimistic. To make the setting more realistic, the vision of the dog should be limited which is parallel to the realities of navigation, crowd control and planning [56–58].

One more extension is to introduce the concept of curriculum learning [59]. Curriculum learning is a technique in machine learning that divides a task into subtasks and gradually increases the difficulty of each subtask. In fact, this idea is frequently used in animal training [60,61]. In the case of our research, an easier starting point might be to learn with a small number of sheep and then gradually increase the number. Alternatively, we can start with a few sheep and a small variance to teach the dog to perform the drive manoeuvre. A task of increased difficulty would then include more sheep in a more dispersed initial state.

Closely related to imitation learning is the concept of inverse reinforcement learning (IRL) [62,63]. Assuming that the expert behaves optimally to maximise its expected reward, the reward could be estimated from the behaviour. This could be an alternative route given that expert demonstrations are available from our simulations.

This work on collective behaviour can give us insights into how the different species of animals perform herding tasks. We have shown that using the models originally meant for herding sheep, we are able to modify these models and apply to other species. We see from both our studies that despite being both herding tasks, there are various subtle differences that arise. Finally, it would be best to examine the meanings between the neural network parameters from an ethological perspective.

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To the Government of Japan,
To Ikeda Sensei,
To Yoshimoto Sensei,
To Nishanth,
To my family,
To all my friends,
It has been a massively wild ride, but it is all worth it.
No amount of thanks will ever be enough.

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[1] Clark Kendrick Go, Monamie Ringhofer, Bryan Lao, Takatomi Kubo, Shinya Yamamoto, Kazushi Ikeda. A mathematical model of herding in a horse-harem group. Behavioural Processes. Submitted.

[2] Clark Kendrick Go, Nishanth Koganti, Kazushi Ikeda. Shepherd-dog model employing imitation learning. IEEE Transactions on Neural Networks and Learning Systems. Submitted.

[3] Monamie Ringhofer, Clark Kendrick Go, Sota Inoue, Renata Mendoça, Satoshi Hirata, Takatomi Kubo, Kazushi Ikeda, Shinya Yamamoto. Herding mechanisms to maintain the cohesion of a harem group: two interaction phases during herding. Journal of Ethology. Submitted.

International Conferences

[1] Clark Kendrick Go, Monamie Ringhofer, Bryan Lao, Takatomi Kubo, Shinya Yamamoto, Kazushi Ikeda. Modelling the Herding of Garrano Horses in the Wild. 2019 Annual Meeting Society for Mathematical Biology, 22 – 26 July, 2019.

[2] Monamie Ringhofer, Clark Kendrick Go, Sota Inoue, Renata S. Mendonça, Satoshi Hirata, Takatomi Kubo, Kazushi Ikeda, Shinya Yamamoto. Herding mechanisms to maintain the cohesion of a harem group: two interaction phases during herding. International Wild Equid Conference 2019, 1 – 5 September, 2019. Submitted.

[3] Monamie Ringhofer, Clark Kendrick C Go, Sota Inoue, Renata S Mendonça, Satoshi Hirata, Takatomi Kubo, Kazushi Ikeda, Shinya Yamamoto. Decision making of mares reacting to the stallion's herding in a harem group of feral horses (*Equus Caballus*). International Society for Behavioral Ecology, 11 – 16 August, 2018.

Local Conferences and Symposia

[1] リングホーファー 萌奈美, Clark Kendrick C. Go, 越智 咲穂, 井上 漱太, Renata S. Mendonça, 平田 聡, 久保 孝富, 池田 和司, 山本 真也. 野生ウマ群のハーディング行動におけるオスとメスの行動調整. The 62nd Primates Conference, 27 – 28 January, 2018.

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