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# Doctoral Dissertation

# Verification Methods for Security against Inference Attacks on XML and Relational Databases

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# Verification Methods for Security against Inference Attacks on XML and Relational Databases∗

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#### Abstract

Access control is one of the most important mechanisms of a database management system to keep confidential information secret, which should be protected from malicious users (or attackers). A typical way of realizing access control is to divide the available set of queries into authorized queries and unauthorized ones, and to allow a user to obtain only the result of authorized queries for a database instance. At first glance, this access control policy works well. However, it may happen that a user can obtain the result (or a set of candidates of the correct result) of an unauthorized query by cleverly combining the results of authorized queries. Such an attack is called an inference attack. This dissertation presents two studies of verifying the security against inference attacks on both XML and relational databases.

Firstly, we study a static analysis problem on k-secrecy, which is a metric for the security against inference attacks on  $\mathbf{XML}$  databases. Intuitively,  $k$ -secrecy means that the number of candidates of sensitive data of a given database instance or the result of an unauthorized query cannot be narrowed down to  $k-1$  by using available information such as authorized queries and their results. We investigate the decidability of the schema-level k-secrecy problem defined as follows: for a given XML database schema, an authorized query and an unauthorized query decide whether every database instance conforming to the given schema is ksecret. We first show that the schema-level k-secrecy problem is undecidable for

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any finite  $k > 1$  even when queries are represented by a simple subclass of linear deterministic top-down tree transducers (LDTT). Next, we give an algorithm for deciding the schema-level ∞-secrecy problem and analyze its time complexity. We show the schema-level  $\infty$ -secrecy problem is EXPTIME-complete for LDTT. Moreover, we show similar results for LDTT with regular look-ahead.

Secondly, we propose a notion of the security against inference attacks by extending  $\ell$ -diversity in relational databases with access control for queries. More specifically, we propose a new privacy notion called *query-based*  $\ell$ *-diversity*. A database instance T is (query-based)  $\ell$ -diverse with respect to given authorized queries if an attacker cannot narrow down the number of possible values of the sensitive information to less than  $\ell$  by inference using the result of the authorized queries on the instance T and the meaning of the queries. We provide two approaches to deciding the property. The first approach directly decides the problem for a given input by using a relational database management system, e.g. Structured Query Language (SQL). The second approach transforms a given input to a logical formula and decides the problem by model counting using a #SAT solver. We discuss the effectiveness and scalability of two approaches based on the experimental results.

#### Keywords:

Database Security, Privacy, Diversity, Inference Attacks, Relational Databases, XML Databases.

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# Chapter 1

# Introduction

Database security is one of the most important challenges in minimizing the leakage of sensitive information due to accesses or to data publishing, which have been growing rapidly and more powerful. Databases may store large amount of sensitive information of individuals or organizations. The problem occurring is that data publishing is vulnerable to attacks, which can lead to serious problems such as privacy information leakage. Database security is built upon a framework of three structures: confidentiality, integrity and availability [2]. Confidentiality refers to the protection of data against an unauthorized access. Integrity refers to the prevention of unauthorized modification. Availability refers to the prevention and recovery from hardware and software errors as well as from malicious data access resulting in the denial of data availability.

# 1.1. Research Motivation

Access control is a means to allow or deny users to do operations such as read, write, modify, etc. on data in a computer system. The system decides whether to allow or deny a user's access requests to information or resources in the system, and if a request is allowed, then that user can do operations according to the access control rules and roles of the user. Access control rules are determined based on information rights, security policy, authorization rules and other factors depending on the system environment. Furthermore, access control is a traditional mechanism for confidentially restricting accesses to a database made by a user by

dividing the queries into authorized and unauthorized ones, and restricting the portion of the data that can be retrieved and updated by the user. However, it may happen that a user can obtain the result (or a set of candidates of correct result) of an unauthorized query by cleverly combining the results of authorized queries. Such an attack is called an inference attack as described below.

Even though a database management system (DBMS for short) provides an appropriate access control mechanism, there is a possibility that sensitive information (unauthorized by access control) can be indirectly obtained from available information (authorized by access control). Let us take a relational database instance r with a schema (student name, year, salary, TA subject) as an example and assume that two queries  $q_1(r) = \pi(\text{student_name}, T\text{A}.\text{subject})(r)$  and  $q_2(r) = \pi(\text{salary}, \text{TA\_subject})(r)$  are authorized and  $q_3(r) = \pi(\text{student\_name},$ salary, TA\_subject $(r)$  is not authorized (because the income of an individual is private information) where  $\pi(X)(r)$  means the projection of r onto X. However, if we take the natural join of  $q_1(r)$  and  $q_2(r)$ , then we can obtain a superset of  $q_3(r)$ , which is a candidate set of the result of the unauthorized query  $q_3$  on r. As shown in this example, an inference attack is a way to infer sensitive information from available information such as the result of authorized queries, the code (meaning) of the authorized and unauthorized queries, and possibly, some external information. If the number of candidates of the result of an unauthorized query is small, then the attacker may obtain information sufficient for his purpose. In the above example, if there are only two candidates (Alice, 100, Database) and (Alice, 70, Database), the attacker knows that the salary of Alice for Database is at least 70.

As shown in the above example, the number of candidates of the sensitive information depends on how secure the system should be. We need a quantitative notion to measure the security or privacy of the system under consideration. There are a few well-known quantitative notions for database privacy, k-anonymity [19][21],  $\ell$ -diversity [15] and t-closeness [14]. These notions assume the following basic concepts on relational databases. The set of attributes are divided into sensitive and nonsensitive attributes. Also, a subset of the nonsensitive attributes, called the quasi-identifier, is assumed. The value of the quasi-identifier is potentially used to identify the tuple of a target individual by linking the disclosed information with external data. A database instance satisfies k-anonymity if for any value of the quasi-identifier, there are  $k$  or more tuples having that value of the quasi-identifier. However, a  $k$ -anonymous database may still have some issues because the database may lack the diversity in the sensitive attributes.  $\ell$ -diversity has been proposed by [15] to overcome the weakness of k-anonymity. Though [15] proposes a general definition of  $\ell$ -diversity, we just review a simple and frequently used one, called distinctive non-recursive  $\ell$ -diversity (or simply,  $\ell$ -diversity). A database instance satisfies  $\ell$ -diversity if for each equivalence class, all tuples within the same class have the same value of the quasi-identifier, there are at least  $\ell$  different values of the sensitive attributes.

Recently, Hashimoto, et al. [11] proposed a quantitative security notion called instance-level k-secrecy to quantify the security against inference attacks on XML databases. Assume that a database instance  $T$  of a schema  $\bf R$ , authorized queries  $q_1, q_2, \ldots, q_m$  and an unauthorized query  $q_s$  are given. An attacker knows **R**,  $q_1, q_2, \ldots, q_m, q_s$  (the meaning of the authorized and unauthorized queries) and  $q_1(T), q_2(T), \ldots, q_m(T)$  (the result of the authorized queries), but he does not know T. The goal of the attacker is to obtain  $q_s(T)$ , which is the result of the unauthorized query  $q_s$  on T (as shown in Example 1.1). For a positive integer k, a database instance T is k-secret with respect to  $\mathbf{R}, q_1, \ldots, q_m, q_s$  if the attacker cannot narrow down the number of the candidates of  $q_s(T)$  to less than k. T is  $\infty$ -secret if the candidates of  $q_s(T)$  are infinite.

Example 1.1.1. Figure 1.1 shows the scenario that an attacker infers the sensitive information  $q_s(T)$  (on the lower left) of a database instance T by combining the result of queries  $q_1(T), q_2(T), ..., q_m(T)$  that gives the candidate set of sensitive information  $q_s(T)$  (on the lower right), which means one of the candidates included in this set is the correct sensitive information which is  $q_s(T)$  (on the lower left). Assume that  $q_1, q_2, ..., q_m$  are authorized queries and  $q_s$  is an unauthorized query.



Figure 1.1. An inference attack.

[11] showed that k-secrecy is decidable for XML databases where queries are given as tree transducers in a certain subclass that can use relabeling and deletion.

## 1.2. Research Contribution

The main purpose of this dissertation is to propose methods for verifying security against inference attacks on XML and relational databases. The following subsections give the summary of the contribution of this thesis.

### 1.2.1 Contribution on XML Databases

To extend the idea of instance-level k-secrecy [11], we introduce the notion of schema-level k-secrecy. For a positive integer  $k$ , a schema **R** is k-secret with respect to authorized queries  $q_1, ..., q_m$  and an unauthorized query  $q_s$  if every instance conforming to  $\bf{R}$  is k-secret with respect to those queries. If we can guarantee that every instance is k-secret, then we do not need to repeatedly test whether a new instance is k-secret or not every time an instance is updated. In this study, we restrict the number of authorized queries to one for simplicity. We show that the schema-level k-secrecy is undecidable for any finite  $k$  when queries are given by linear deterministic top-down tree transducers. Also, we show that the schema-level  $\infty$ -secrecy is decidable for the same class of queries and investigate the computational complexity of the problem. The details of this study will be discussed in Chapter 3.

#### 1.2.2 Contribution on Relational Databases

To apply the idea of instance-level  $k$ -secrecy [11] in defining a quantitative notion for anonymity, this study introduces a notion of the security against inference attacks by extending  $\ell$ -diversity [15] in relational databases with access control for queries. More specifically, we propose a new privacy notion called *query*based  $\ell$ -diversity. A database instance T is (query-based)  $\ell$ -diverse with respect to given authorized queries if an attacker cannot narrow down the number of possible values of the sensitive information for any individual to less than  $\ell$  by inference, based on the result of the authorized queries on the instance  $T$  and the queries themselves. Two approaches are proposed and implemented by Structured Query Languages (SQL) and #SAT solver, respectively. Also, we compare the scalability and effectiveness of the two approaches based on the experiments conducted on the implemented tools. The first approach is based on relational algebra and restricts the authorized queries only on projection queries (selection is not allowed). The second approach transforms a given input of the problem to a logical formula and decides the problem by model counting using #SAT solver, where queries are self-join free conjunctive queries, consisting of projection, selection and join without self-join. The details of this study will be presented in Chapter 4.

### 1.3. Dissertation Layout

This dissertation is organized as follows:

Chapter 1 presents the purpose by introducing the motivations of this dissertation. Mainly, k-secrecy, the verification notion on XML databases is discussed. Also, the research contributions are briefly introduced to both XML and relational databases. Chapter 2 presents the related security and privacy notions in details. Chapter 3 extends the notion of instance-level k-secrecy to schemalevel k-secrecy. The decidability and computational complexity of the problem of schema-level k-secrecy are investigated for the class of queries represented by linear deterministic top-down tree transducers. Chapter 4 gives the proposal of the query-based  $\ell$ -diversity, which is a notion obtained by combining k-secrecy and  $\ell$ -diversity in the relational database setting. We propose two approaches to the problem of verifying the query-based  $\ell$ -diversity. One of them based on relational algebra and the other one is based on model counting. Then we end with the experimental results conducted on two verification tools implemented by SQL and #SAT solver. Chapter 5 provides the conclusion of this dissertation.

# Chapter 2

# Related Work

In the introduction, we briefly discussed about an inference attack, which is the main problem of this dissertation. Furthermore, we also mentioned access control as a traditional mechanism that confidentially restricts accesses to a database. Finally, we discussed about security and privacy notions such as k-anonymity,  $\ell$ -diversity and k-secrecy. This chapter presents these notions in more details.

## 2.1.  $k$ -Anonymity

A database instance satisfies k-anonymity if for any value of the quasi-identifier, there are  $k$  or more tuples having that value of the quasi-identifier. A maximal subset of tuples having same values of the quasi-identifier is called an equivalence class. k-anonymity means that the cardinality of each equivalence class is at least  $k$ . A transformation of a given instance to another instance satisfying k-anonymity is called a k-anonymization for the original instance. [21] proposed a method for k-anonymization by hiding some information of individuals by generalization and suppression. Generalization replaces a value with a less specific but semantically consistent value, while suppression hides the data or does not release the entire data. Various anonymization methods have been reported using clustering, branch-and-bound search and so on  $[1][4]$ . k-anonymity is a simple notion and has been frequently used.

k-anonymity guarantees that an individual's information cannot be distinguished from a given number of people in the record of size  $k$ . In general, the higher the value of  $k$ , the more privacy is achieved.

Example 2.1.1. Consider a database instance in Table 2.1, which contains two attributes; Zipcode and Age. The instance is k-anonymous if for any record,  $k-1$  other records have the same Zipcode and Age. Table 2.1 satisfies 2-anonymity.

Zipcode	Age
$130$ <sup>****</sup>	$2^*$
$130-***$	$2*$
$130$ -****	$2*$
$130$ -****	$2^*$
$630-***$	$2*$
$630-***$	$2^*$

Table 2.1. A 2-anonymous instance.

As discussed in [15], however, a k-anonymous database may still have some issues because the database may lack the diversity in the sensitive attributes.  $\ell$ -diversity has been proposed by [15] to overcome the weakness of k-anonymity.

# 2.2.  $\ell$ -Diversity

 $\ell$ -diversity has been proposed by [15] to overcome the weakness of k-anonymity and improves anonymization beyond what  $k$ -anonymity provides. [24] showed that  $\ell$ -diversity always guarantees stronger privacy preservation than  $k$ -anonymity. The difference between these two notions is that while k-anonymity requires each equivalence classes of quasi-identifier to have k entires,  $\ell$ -diversity requires that there are  $\ell$  different sensitive values for each quasi-identifier, as shown in Table 2.3. Though [15] proposes a general definition of  $\ell$ -diversity, we review a simple and frequently used one, called distinctive non-recursive  $\ell$ -diversity (or simply,  $\ell$ -diversity). A database instance satisfies  $\ell$ -diversity if for each equivalence class, there are at least  $\ell$  different values of the sensitive attributes. Distinctive  $\ell$ diversity shows that sensitive values are distinctive from each other for at least  $\ell$ values in the same equivalence class.

Example 2.2.1. Table 2.2 shows a database instance consisting of six tuples. Assume that {Zipcode, Gender, Age} is the quasi-identifier and Diagnosis is the sensitive attribute. All the tuples have different values of the quasi-identifier and hence this database instance does not satisfy  $k$ anonymity for any  $k \geq 2$ .

Assume that in this database instance, the latter four digits of the values of "Zipcode" are hidden, the values of "Gender" are hidden and the values of "Age" are generalized to the intervals of ten years. Then we obtain the database instance shown in Table 2.3. This instance consists of two equivalence classes and the number of tuples in each class is three. Hence, this instance satisfies 3-anonymity and the above transformation is a 3-anonymization for the original instance. Also, the first class has three different values of the sensitive attribute and the second class has two different values. Hence, the transformed instance satisfies 2-diversity but does not satisfy  $\ell$ -diversity for any  $\ell \geq 3$ .

$10000$ $\mu$ . If both promotions.					
Zipcode	Gender	Age	Diagnosis		
123-4567	F	45	А		
123-5235	$_{\rm F}$	44	В		
123-4567	$_{\rm F}$	44	$\mathcal{C}$		
378-2102	М	65	А		
378-2102	M	62	B		
378-2102	F	65			

Table 2.2. A sample instance.





### 2.3.  $k$ -Secrecy

A related but different quantitative notion on database security is given in [11] based on access control on queries. Assume that a database instance T of a schema **R**, authorized queries  $q_1, q_2, \ldots, q_m$  and an unauthorized query  $q_s$  are given. An attacker knows  $\mathbf{R}, q_1, q_2, \ldots, q_m, q_s$  (the meaning of the authorized and unauthorized queries) and  $q_1(T), \ldots, q_m(T)$  (the result of the authorized queries), but he does not know T. The goal of the attacker is to obtain  $q_s(T)$ , which is the result of the unauthorized query  $q_s$  on T. For a positive integer k, a database instance T is k-secret with respect to  $\mathbf{R}, q_1, \ldots, q_m, q_s$  if the attacker cannot narrow down the number of the candidates of  $q_s(T)$  to less than k. T is  $\infty$ -secret if the candidates of  $q_s(T)$  are infinite. [11] showed that k-secrecy is decidable for XML databases where queries are given as tree transducers in a certain subclass that can use relabeling and deletion. Example 2.3.1 shows the example of an inference attack on an XML database instance.

Example 2.3.1. Figure 2.1 illustrates the example of an inference attack on an XML database instance. Assume that an attacker wants to know the salary (sensitive information of an individual) of an employee named Taro, whose lives in Nara, but an attacker does not have permission to access to such information. Meanwhile, there are some available information which has been retrieved by authorized queries such as the results of queries  $q_1$ (a pair of "Name" and "Address"), and q<sup>2</sup> (a pair of "Salary" and "Address"). An attacker obtains the sensitive information by combining these available information and narrowing down candidate set of sensitive information  $Y(t)$ . An attacker knows that one of the candidates set  $Y(t)$  is the correct information of Taro or the salary of Taro is at least \$1000.



Figure 2.1. An attack on XML instance.

Although  $[11]$  deals with XML databases, the notion of k-secrecy is general enough for other kinds of databases. Furthermore, [10] discussed verification of the security against inference attacks on XML databases, considering a functional dependency when unauthorized query is represented by a deterministic top-down tree transducer. More sophisticated notions have been also proposed. For example, [6][16] proposed stronger notions where the probability distribution of possible secrets does not change after observing (authorized) information.

# Chapter 3

# Deciding Schema *k*-Secrecy on XML Databases

This chapter presents the notion of schema-level k-secrecy, which is the extension of instance-level k-secrecy. The definition and restriction of the problem are presented. Also, the decidability results and its computational complexity are discussed.

# 3.1. Introduction

Access control is one of the most important mechanisms of a database management system to keep confidential information secret, which should be protected from malicious access. A typical way of realizing access control is to divide the available set of queries into authorized queries and unauthorized ones and to allow a user to obtain only the result of the authorized queries for a database instance.

At first glance, this access control policy works well. However, it may happen that a user can obtain the result (or a set of candidates of the correct result) of an unauthorized query by cleverly combining the results of authorized queries. Such an attack is called an inference attack. Hence, it is desirable to be able to decide whether an inference attack is possible or not for a given database. To quantify the degree of safeness against inference attacks, Hashimoto, et al. [11] define ksecrecy. Intuitively, a database instance is k-secret if an attacker cannot narrow down the number of candidates of the result of an unauthorized query to less

than k. In particular, an instance is  $\infty$ -secret if the number of candidates cannot be narrowed down to a finite value. The underlying idea is similar to  $\ell$ -diversity in [15]. For a given XML database schema  $A_G$ , a database instance t conforming to the schema, an authorized query  $T_a$  and an unauthorized query  $T_s$ , we say that t is k-secret if  $|Y(t)| \geq k$  where  $Y(t) = \{T_s(t') | t' \in L(A_G), T_a(t') = T_a(t)\}\$ and  $L(A_G)$  is the set of instances conforming to  $A_G$ . In particular, t is  $\infty$ -secret if  $Y(t)$  is infinite.  $A_G$  is given by a tree automaton, and queries  $T_a$  and  $T_s$  are given by tree transducers. In [11], it is shown that  $k$ -secrecy is decidable and its time complexity is polynomial in the size of a given instance  $t$  if each query is represented by a composition of relabeling and/or deleting tree transducers. The problem discussed in [11] is in instance-level in the sense that we are required to decide whether a given instance t is k-secret or not. In this study, we discuss the decidability of the problem for *schema-level k-secrecy* (or simply schema ksecrecy), which is the problem of deciding whether *every instance* conforming to a given schema is  $k$ -secret. If we can guarantee that every instance is  $k$ -secret, then we do not need to repeatedly test whether a new instance is k-secret or not every time an instance is updated.

### 3.2. Models

#### 3.2.1 XML Documents

XML documents or XML database instances are often modeled by *unranked la*beled ordered trees, where each node has an arbitrary number of child nodes. Moreover, we do not consider attributes as the input of  $k$ -secrecy problem. In this chapter, for simplicity, we consider only *binary* labeled ordered trees. Some encodings from unranked trees to binary trees, without loss of expressive power, are proposed [5][9]. Among them, we adopt *First-Child-Next-Sibling* encoding [9], which is a simple and widely accepted encoding. The first child of a node in an unranked tree corresponds to the left child of the node in the encoded binary tree, and the next sibling of a node in an unranked tree corresponds to the right child of the node in the encoded binary tree. We use  $\#$  as the special symbol which means that there is no child or no next sibling.

The set  $\mathcal{T}_{\Sigma}$  of (binary) labeled ordered trees over an alphabet  $\Sigma$  is the smallest set defined by:

- $\bullet \# \in \mathcal{T}_{\Sigma}$ , and
- $a(t_1, t_2) \in \mathcal{T}_{\Sigma}$  if  $t_1, t_2 \in \mathcal{T}_{\Sigma}$  and  $a \in \Sigma$ .

For a tree  $t \in \mathcal{T}_{\Sigma}$ , let  $V(t)$  denote the set of nodes of t, and let  $\lambda(v)$  denote the label of a node  $v \in V(t)$ . For  $t, t' \in \mathcal{T}_{\Sigma}$  and  $v \in V(t)$ , let  $t/v$  denote the subtree of t rooted at v and let  $t[v \leftarrow t']$  denote the tree obtained from t by replacing  $t/v$  by  $t'$ . The size of a tree t is the number of the nodes of t. Let  $\mathcal{X}_2$  be the set consisting of two variables  $x_1$  and  $x_2$ . A *context* C over  $\Sigma$  is a tree over  $\Sigma \cup \mathcal{X}_2$  such that  $x_1$  and  $x_2$  can only appear on leaves of the tree at most once, respectively. Let  $\mathcal{C}(\Sigma, \mathcal{X}_2)$  be the set of all contexts over  $\Sigma$ . For a context  $C \in \mathcal{C}(\Sigma, \mathcal{X}_2)$  and  $t_1, t_2 \in \mathcal{T}_{\Sigma}$ , let  $C[t_1, t_2]$  denote the tree in  $\mathcal{T}_{\Sigma}$  obtained from C by replacing each variable  $x_i$  by  $t_i$  for  $i \in \{1, 2\}.$ 

#### 3.2.2 XML Databases

A database schema describes syntactic restriction on database instances. Schema for XML documents are modeled by tree automaton.

**Definition 3.2.1.** A tree automaton (TA) A is a 4-tuple  $(Q, \Sigma, Q_0, \delta)$ , where Q is a finite set of states,  $\Sigma$  is an alphabet,  $Q_0 \subseteq Q$  is the set of initial states, and  $\delta$  is a set of transition rules in the form of either  $q \to a(q_1, q_2)$  or  $q \to \#$ , where  $q, q_1, q_2 \in Q$  and  $a \in \Sigma$ .

A tree  $t \in \mathcal{T}_{\Sigma}$  is accepted by A if and only if there is a mapping r from  $V(t)$ to Q such that

- $r(v_0) \in Q_0$  where  $v_0$  is the root of t
- $r(v) \rightarrow \lambda(v)(r(v_1), r(v_2)) \in \delta$  if  $v_1$  and  $v_2$  are the left and right children of v, respectively and
- $r(v) \rightarrow \# \in \delta$  if v is a leaf.

The mapping r is called a *successful run* of A on t. A successful run will be called a run in this study. We define the language  $L(A)$  to be  $\{t \in \mathcal{T}_{\Sigma} \mid A\}$ accepts  $t \}$ .  $L(A)$  is called the language recognized by A. A regular tree language is a language recognized by a TA. The size of a TA A is the number of the states and transition rules of A. A TA A is unambiguous if for every  $t \in L(A)$ , a run of A on t is unique.

For a TA  $A = (Q, \Sigma, Q_0, \delta)$ , define the dependency graph of A as the directed graph  $G_A = (Q, E_A)$  where  $E_A = \{(q, q') | \exists q'' \in Q, \exists a \in \Sigma, q \rightarrow a(q', q'') \in \delta\}$ or  $q \to a(q'', q') \in \delta$ . A path of a directed graph is nonempty if it consists of at least one edge. For a TA  $A = (Q, \Sigma, Q_0, \delta)$ , we say that a state  $q \in Q$  is *recursive* through a nonempty path of nodes (states)  $q_1, q_2, \ldots, q_n$   $(n \geq 2)$  of  $G_A$  such that  $q_1 = q_n = q.$ 

#### 3.2.3 Queries

As languages for XML queries/transformations, XQuery and XSLT are developed and recommended by W3C (World Wide Web Consortium). Both queries and programs written in XQuery and XSLT can be regarded as transformations from trees to trees. Unfortunately, many static analysis problems are undecidable for the full class because XQuery and XSLT are Turing complete [12]. The core of tree transduction of XSLT can be modeled by tree transducers [3][5][9]. Tree transducers are a finite machine model for tree transformations. As query models, we use linear deterministic top-down tree transducers (LDTT) and linear deterministic top-down tree transducers with regular look-ahead  $(LDTT<sup>R</sup>)$ .

**Definition 3.2.2.** A *linear top-down tree transducer* (LTT)  $T$  is a quintuple  $(Q, \Sigma, \Delta, Q_0, \delta)$ , where Q is a finite set of states,  $\Sigma$  is an input alphabet,  $\Delta$  is an output alphabet,  $Q_0 \subseteq Q$  is the set of initial states, and  $\delta$  is a set of transduction rules of the following two types:

$$
q(a(x_1, x_2)) \to C[q_1(x_1), q_2(x_2)]
$$
 or  $q(\#) \to t'$ 

where  $a \in \Sigma$ ,  $q, q_1, q_2 \in Q$ ,  $C \in \mathcal{C}(\Delta, \mathcal{X}_2)$ , and  $t' \in \mathcal{T}_{\Delta}$ .

The move relation  $\rightarrow$  of T is defined as follows:

• If  $q(a(x_1, x_2)) \to C[q_1(x_1), q_2(x_2)] \in \delta$ ,  $t_1, t_2 \in \mathcal{T}_{\Sigma}$  and  $t/v = q(a(t_1, t_2))$ then,  $t \to t[v \leftarrow C[q_1(t_1), q_2(t_2)]]$  and

• If 
$$
q(\#) \to t' \in \delta
$$
 and  $t/v = q(\#)$  then  $t \to t[v \leftarrow t']$ .

The reflexive and transitive closure of  $\rightarrow$  is denoted by  $\rightarrow^*$ . The transformation induced by T, also denoted as T, is the relation defined by:  $T = \{(t, \tau) \mid q_0(t) \to^*$  $\tau, t \in \mathcal{T}_{\Sigma}, \tau \in \mathcal{T}_{\Delta}, q_0 \in Q_0$ . The domain  $Dom(T)$  of T is the set  $\{t \in \mathcal{T}_{\Sigma} \mid \exists \tau \in \mathcal{T}_{\Sigma}\}$  $\mathcal{T}_{\Delta}.(t,\tau) \in T$ . The image of a subset  $L \subseteq \mathcal{T}_{\Sigma}$  by T is the set  $T(L) = \{ \tau \in \mathcal{T}_{\Delta} \mid$  $\exists t \in L(t, \tau) \in T$ . The size of an LTT T is the sum of the number of the states and the sizes of the transduction rules of T. The size of a transduction rule  $\rho$  is the size of the context in the right-hand side of  $\rho$ . An LTT  $T = (Q, \Sigma, \Delta, Q_0, \delta)$ is *deterministic* (denoted LDTT) if (1)  $Q_0$  is a singleton and (2) for each state q and  $a \in \Sigma$ , there is at most one rule that contains q and a in its left-hand side. When T is deterministic, we can regard T as a function from  $Dom(T)$  to  $\mathcal{T}_{\Delta}$ . Also we let  $T^{-1}(t') = \{t \mid T(t) = t'\}.$ 

The regular look-ahead [7] allows that, when processing each node in topdown manner, it checks whether the subtrees rooted by its children are contained in specified regular tree languages and then processes the node if the containment holds. This ability of look-ahead corresponds to the XPath filters used in XSLT programs.

Definition 3.2.3. A linear top-down tree transducer with regular look-ahead (LTTR) T is a quintuple  $(Q, \Sigma, \Delta, Q_0, \delta)$ , where Q is a finite set of states,  $\Sigma$ is an input alphabet,  $\Delta$  is an output alphabet,  $Q_0 \subseteq Q$  is the set of initial states, and  $\delta$  is a set of transduction rules of the following two types:

$$
q(a(x_1 : A_1, x_2 : A_2)) \to C[q_1(x_1), q_2(x_2)]
$$
 or  
 $q(\#) \to t'$ 

where  $a \in \Sigma$ ,  $q_1, q_2, q \in Q$ ,  $C \in \mathcal{C}(\Delta, \mathcal{X}_2)$ ,  $t' \in \mathcal{T}_{\Delta}$ , and  $A_1, A_2$  are TAs over  $\Sigma$ .

The move relation  $\rightarrow$  of T is defined as follows:

•  $t \to t[v \leftarrow C[q_1(t_1), q_2(t_2)]]$  if  $q(a(x_1 : A_1, x_2 : A_2)) \to C[q_1(x_1), q_2(x_2)] \in \delta$ ,  $t_1 \in L(A_1), t_2 \in L(A_2)$  and  $t/v = q(a(t_1, t_2))$  and

• 
$$
t \to t[v \leftarrow t']
$$
 if  $q(\#) \to t' \in \delta$  and  $t/v = q(\#)$ .

The transformation induced by  $T$  is defined in the same way as LTT. The size of an  $LTT^{R}T$  is the sum of the number of the states and the sizes of the transduction rules of T. The size of a transduction rule  $\rho$  is the sum of the size of the context in the right-hand side and the sizes of the TAs in the left-hand side. An  $LTT^{R}$ T is deterministic (denoted LDTT<sup>R</sup>) if (1)  $Q_0$  is a singleton and (2) there are no two different rules  $q(a(x_1 : A_1, x_2 : A_2)) \to \rho_1$  and  $q(a(x_1 : A'_1, x_2 : A'_2)) \to \rho_2$ such that  $L(A_1) \cap L(A'_1) \neq \emptyset$  and  $L(A_2) \cap L(A'_2) \neq \emptyset$ . It is known that the class of transformations induced by  $LDTT^R$  is a superclass of those by LDTT.

### 3.3. Schema k-Secrecy Problem

**Definition 3.3.1** (*k*-Secrecy). Given a database schema  $A_G$ , a database instance t, an authorized query  $T_a$  and an unauthorized query  $T_s$  where  $Dom(T_a) \supseteq L(A_G)$ and  $Dom(T_s) \supseteq L(A_G)$ . We say that  $(t, T_a, T_s)$  is k-secret if  $|Y(t)| \geq k$  where  $Y(t) = \{T_s(t') \mid t' \in L(A_G), T_a(t') = T_a(t)\}.$  In particular,  $(t, T_a, T_s)$  is  $\infty$ -secret if  $Y(t)$  is infinite. If  $T_a$  and  $T_s$  are clear from context, we simply say t is  $k(\infty)$ -secret.

In the above definition,  $Y(t)$  represents the candidates of secret information. We assume that attackers know the schema, the definition of both authorized and unauthorized queries as well as the results of authorized queries applied to  $t$ , but do not know the instance t and the secret information  $T_s(t)$ . Note that  $Y(t)$  can be represented as  $Y(t) = T_s(T_a^{-1}(T_a(t)) \cap L(A_G))$ . An attacker uses the available information from the result of authorized queries  $T_a(t)$  to compute the inverse image  $T_a^{-1}(T_a(t))$ , then extracts from  $T_a^{-1}(T_a(t))$  the trees that conform to the given schema  $A_G$ . After that, the attacker executes the unauthorized query  $T_s$ in order to obtain the candidate secret information  $Y(t)$  of the database.  $Y(t)$  is the set consisting of all candidates that the attacker cannot distinguish from the secret information  $T<sub>s</sub>(t)$ .

Note that a database instance always satisfies 1-secrecy and hence when we refer to k-secrecy, we implicitly assume that  $k \geq 2$ .

**Definition 3.3.2** (Schema k-Secrecy). Given a database schema  $A_G$ , an authorized query  $T_a$ , and an unauthorized query  $T_s$ ,  $(A_G, T_a, T_s)$  is schema k-secret if every instance  $t \in L(A_G)$  is k-secret.

Schema k-secrecy (or schema  $\infty$ -secrecy) problem, abbreviated as k-SS (or  $\infty$ -SS), asks whether, given a database schema  $A_G$ , an authorized query  $T_a$ , and an unauthorized query  $T_s$ ,  $(A_G, T_a, T_s)$  is schema k-secret (or schema  $\infty$ -secret). In this study, we assume the number of authorized queries is one and leave the extension to multiple authorized queries as future work (see Section 3.6.1).

**Example 3.3.3.** Consider TA  $A_G$  and transducers  $T_a^1, T_a^2, T_s^1$  and  $T_s^2$  defined as follows:

- $\bullet \Sigma = \{a, \tilde{a}, s, \# \}, Q = \{q_0, q_1, q_* \} \text{ and } A_G = (Q, \Sigma, \{q_0\}, \{q_0 \to a(q_1, q_*),$  $q_0 \to \tilde{a}(q_1, q_*)$ ,  $q_1 \to s(q_1, q_*)$ ,  $q_1 \to s(q_*, q_*)$ ,  $q_* \to \#$ ).  $A_G$  accepts the trees which have the shapes shown in Figure 3.1.
- $T_a^i = (\{q\}, \Sigma, \Sigma, \{q\}, \delta_a^i)$   $(i = 1, 2)$  where  $\delta_a^1 = \{q(a(x_1, x_2)) \rightarrow a(q(x_1), q(x_2)),$  $q(\tilde{a}(x_1, x_2)) \to a(q(x_1), q(x_2)), q(s(x_1, x_2)) \to q(x_1), q(\#) \to \#$  and  $\delta_a^2$  is the set of rules obtained from  $\delta_a^1$  by replacing  $q(\tilde{a}(x_1, x_2)) \to a(q(x_1), q(x_2))$ with  $q(\tilde{a}(x_1, x_2)) \rightarrow \tilde{a}(q(x_1), q(x_2))$ . Both  $T_a^1$  and  $T_a^2$  delete all s and  $T_a^1$ replaces  $\tilde{a}$  with a.
- $T_s^1$  is an LDTT that induces the identity map on  $L(A_G)$ .
- $T_s^2 = (\{q\}, \Sigma, \Sigma, \{q\}, \delta_s^2)$  where  $\delta_s^2 = \{q(a(x_1, x_2)) \to a(\#, \#), q(\tilde{a}(x_1, x_2)) \to a(\#, \#), \delta_s^2\}$  $\tilde{a}(q(x_1), q(x_2)), q(s(x_1, x_2)) \to s(q(x_1), q(x_2)), q(\#) \to \#$ .  $T_s^2$  deletes all s only after it reads a.

For  $m \geq 1$ , let  $as^m$  and  $\tilde{a}s^m$  be the trees of the shape shown in Figure 3.1 (a) and (b) respectively. Also let  $as^+ = \{as^m \mid m \ge 1\}$  and  $\tilde{a}s^+ = \{\tilde{a}s^m \mid m \ge 1\}$ . A symbol a stands for  $a(\#, \#)$  or  $\{a(\#, \#)\}\$  depending on the context. For each  $t \in L(A_G)$ ,  $T_a^1(t) = a$ . Thus,  $(T_a^1)^{-1} \circ T_a^1(t) \cap L(A_G) = L(A_G)$ .  $T_a^2(as^m) = a$  and  $T_a^2(\tilde{a}s^m) = \tilde{a}$ . Thus,  $(T_a^2)^{-1} \circ T_a^2(as^m) \cap L(A_G) = as^+$  and  $(T_a^2)^{-1} \circ T_a^2(\tilde{a}s^m) \cap$  $L(A_G) = \tilde{a}s^+$ . Since  $T_s^1$  is the identity map,  $(A_G, T_a^1, T_s^1)$ ,  $(A_G, T_a^2, T_s^1)$  are schema ∞-secret.  $(A_G, T_a^1, T_s^2)$  is also schema ∞-secret because  $T_s^2(L(A_G)) = a \cup \tilde{a} s^+$ while  $(A_G, T_a^2, T_s^2)$  is not schema  $\infty$ -secret because  $T_s^2(as^+) = a$ . See Table 3.1.



Figure 3.1. Trees accepted by  $A_G$ .

				$\cdot$	<b>I</b>	
$T_s^i$ $T_a^i$	$\boldsymbol{t}$	$T_a^i(t)$	A(t)	Y(t)		$\infty$ -secret
$\mathcal{T}^1_a, \mathcal{T}^1_s$	$as^m$ $\bar{\tilde{a}}\bar{s}^{\bar{m}}$	$\boldsymbol{a}$	$L(A_G)$	$L(A_G)$	$\infty$	yes
$\mathbb{T}^2_a, \mathbb{T}^1_s$	$as^m$ $\overline{\tilde{a}}\overline{s}^m$	$\boldsymbol{a}$ $\tilde{a}$	$as^+$ $\tilde{a}s^+$	$as^+$ $\tilde{a}s^+$	$\infty$	yes
$T_a^1, T_s^2$	$as^m$ $\bar{\tilde{a}}\bar{s}^{\overline{m}}$	$\boldsymbol{a}$	$L(A_G)$	$a \cup \tilde{a} s^+$	$\infty$	yes
$\mathcal{T}^2_a, \mathcal{T}^2_s$	$as^m$ $\bar{\tilde{a}}\bar{s}^{\overline{m}}$	$\boldsymbol{a}$ $\tilde{a}$	$as^+$ $\tilde{a}s^+$	$\boldsymbol{a}$ $\tilde{a}s^+$	1 $\infty$	$\mathop{\mathrm{no}}$

Table 3.1. Schema  $\infty$ -secrecy for Example 3.3.3.

We let  $A(t) = (T_a^i)^{-1} \circ T_a^i(t) \cap L(A_G)$  and  $Y(t) = T_s^i((T_a^i)^{-1} \circ T_a^i(t) \cap L(A_G)).$ Note that  $L(A_G) = as^+ \cup \tilde{a}s^+$ .

## 3.4. Undecidability Result

We show that k-SS is undecidable for any finite  $k > 1$ , even if queries are restricted to deterministic rational transducers on words (DRTW) [5] where a rule has the shape  $p(ax) \rightarrow \alpha q(x)$  (a is an input symbol, x is a variable denoting the remaining input string,  $p, q$  are states and  $\alpha$  is a string of output symbols). DRTW can be regarded as a proper subclass of LDTT.

**Theorem 3.4.1.** k-SS is undecidable for any integer  $k > 1$ , where queries are represented by DRTW.

*Proof.* We show the undecidability of 2-SS by reduction from PCP. The undecidability of k-SS for  $k > 2$  can be shown in almost the same way. Here, we use a regular expression to represent a schema and DRTW to represent queries. The class of languages represented by regular expressions, when regarded as tree languages, is a proper subclass of regular tree languages.

Consider any instance  $(W, U)$  of PCP, where  $W = \langle w_1, w_2, \ldots, w_n \rangle$  and  $U =$  $\langle u_1, u_2, \ldots, u_n \rangle$  are both *n*-tuples of words over an alphabet Σ. Let  $[n] = \{1, \ldots, n\}$ and we assume that  $\Sigma \cap [n] = \emptyset$ . We construct the following regular expression  $A_G$  over  $\Sigma \cup [n]$  as a schema:

$$
A_G = (1w_1 \mid 2w_2 \mid \ldots \mid nw_n)^+ \mid (1u_1 \mid 2u_2 \mid \ldots \mid nu_n)^+.
$$

For example, let  $W = \langle abc, d \rangle$  and  $U = \langle a, bcd \rangle$ , then we construct  $A_G = (1abc \mid$  $2d)^+$  |  $(1a \mid 2bcd)^+$ . Let  $L(A_G)$  be the language represented by  $A_G$ . Next, let  $T_a$ be the authorized query that extracts only symbols in  $[n]$  from an input word, and let  $T_s$  be the unauthorized query that extracts only symbols in  $\Sigma$  from an input word. For example, given  $t = 1abc2d$  as an input word,  $T_a(t) = 12$  and  $T_s(t) = abcd$ . Both  $T_a$  and  $T_s$  can be defined by DRTW with one state.

We now show that  $|T_s(T_a^{-1}(T_a(t)) \cap L(A_G))| \geq 2$  for any  $t \in L(A_G)$  if and only if  $(W, U)$  has no solution. Take any  $t = i_1 w_{i_1} i_2 w_{i_2} \cdots i_m w_{i_m} \in L(A_G)$  (or  $t = i_1 u_{i_1} i_2 u_{i_2} \cdots i_m u_{i_m} \in L(A_G)$ , and then it holds that  $T_a^{-1}(T_a(t)) \cap L(A_G) =$  $\{i_1w_{i_1}i_2w_{i_2}\cdots i_mw_{i_m}, i_1u_{i_1}i_2u_{i_2}\cdots i_mu_{i_m}\}.$  Thus,

$$
Y(t) = T_s(T_a^{-1}(T_a(t)) \cap L(A_G)) = \{w_{i_1}w_{i_2}\cdots w_{i_m}, u_{i_1}u_{i_2}\cdots u_{i_m}\}.
$$

This situation is depicted in Figure 3.2. If the instance  $(W, U)$  of PCP has a solution  $i_1 i_2 \cdots i_m$ , then  $|Y(t)| = 1$  where  $t = i_1 w_{i_1} i_2 w_{i_2} \cdots i_m w_{i_m}$  or  $t = i_1 u_{i_1} i_2 u_{i_2} \cdots i_m u_{i_m}$  because  $w_{i_1} w_{i_2} \cdots w_{i_m} = u_{i_1} u_{i_2} \cdots u_{i_m}$ . Conversely, if the instance  $(W, U)$  of PCP has no solution, then  $|Y(t)| = 2$  for any  $t \in L(A_G)$ because  $w_{i_1}w_{i_2}\cdots w_{i_m}\neq u_{i_1}u_{i_2}\cdots u_{i_m}$  for any  $i_1i_2\cdots i_m\in[n]^+$ . Thus, we have proved that 2-SS is undecidable.  $\Box$ 



Figure 3.2. Reduction from PCP to 2-SS.

## 3.5. Decidability Results on  $\infty$ -SS

In this section, we first give an overview of our decision algorithm for  $\infty$ -SS in Section 3.5.1, and the details of the algorithm where queries are represented by LDTT in Section 3.5.2. Then, we prove the correctness of the algorithm in Section 3.5.3, and analyze the time complexity in Section 3.5.4. The results are extended to  ${\rm LDTT}^R$  in Section 3.5.5.

#### 3.5.1 Overview of Decision Algorithm

For simplicity, we consider only authorized/unauthorized queries but not schemas here. More precisely, we fix a schema to be a TA which accepts any tree. We assume that a schema is arbitrarily given as a TA when we give the detail of our algorithm in Section 3.5.2.

Consider a pair  $(t, T(t))$  of input and output trees of an LDTT T. For each node v of  $t \in Dom(T)$ , v either is transformed to some nonempty part of  $T(t)$  by applying some rule of T to v, or does not correspond to any part of  $T(t)$ , that is, v is deleted by  $T$ . For example, let  $T$  be the LDTT which has the transduction rules listed in Figure 3.3. The left tree is transformed into the right tree in Figure 3.3 by T. The nodes of the left tree in the circle-dot-lines correspond to the parts of the right tree according to the applied rules of  $T$ , respectively. On the other hand, the nodes in the rectangle-dot-line are deleted by T. The subtree  $c(\#, \#)$ is deleted because  $q_1(b(x_1, x_2)) \rightarrow \#$  is applied to the parent node b and thus the subtrees at  $x_1$  and  $x_2$  are ignored (or not touched). Note that the node b



Figure 3.3. Deleted and undeleted nodes by T.

is not deleted but transformed to  $#$ . We call a rule such that a variable in its left hand side does not occur in its right hand side, like  $q_1(b(x_1, x_2)) \rightarrow #$ , a subtree-deleting rule. The node b which is the right child of the root is deleted by T because  $q_2(b(x_1, x_2)) \rightarrow q(x_1)$  is applied to the node and thus no symbol is output by this rule. We call a rule without output symbols in the right hand an erasing rule. Note that any erasing rule is also a subtree-deleting rule because only one variable can occur in its right-hand side of the erasing rule.

We intend to show a decision algorithm for  $\infty$ -secrecy of  $(A_G, T_a, T_s)$  which will be explained later. The unambiguous TA  $A_a$  can be constructed from a query  $T_a$  as follows:

- $A_a$  recognizes the domain of  $T_a$ .
- Each state of  $A_a$  is either a rule  $\rho$  of  $T_a$  or a special state ⊥.
- For any input tree t and any node v of t, if the unique run of  $A_a$  on t assigns a rule  $\rho$  of  $T_a$ , then  $T_a$  applies the rule  $\rho$  at v when t is transformed by  $T_a$ ;

if the run assigns  $\perp$  to v, then  $T_a$  deletes v by applying a subtree-deleting rule at some proper ancestor of v.

For an input tree  $t \in Dom(T_a)$ , in order to distinguish whether a node is deleted by  $T_a$ , we just look at the state assigned to the node of t by  $A_a$ . That is,  $\perp$  or  $\rho$  where  $\rho$  is an erasing rule is assigned to the node if and only if the node is deleted by  $T_a$ . We call such states *deleting states* of  $A_a$ . Similarly, let  $A_s$  denote the unambiguous TA constructed from  $T_s$  in the same way as  $A_a$ .

Our algorithm consists of four steps:

- 1. Construct tree automata  $A_a$  and  $A_s$  from  $T_a$  and  $T_s$  s.t  $L(A_a) = Dom(T_a)$ and  $L(A_s) = Dom(T_s)$ .
- 2. Construct  $A_I = A_a \times A_s \times A_G$ .
- 3. Construct tree automata  $A_{inf}$  and  $A_{fin}$  from  $A_I$  such that
	- $L(A_{fin}) \cup L(A_{inf}) = L(A_I), L(A_{fin}) \cap L(A_{inf}) = \emptyset$  and
	- $T_a(L(A_{fin})) \subset T_a(L(A_{inf}))$  iff  $(A_G, T_a, T_s)$  is  $\infty$ -secret

by identifying the states of  $A_I$  which can occur recursively and contain deleting states of  $A_a$  and non-deleting states of  $A_s$  as their components.

4. Decide whether  $T_a(L(A_{fin})) \subseteq T_a(L(A_{inf})).$ 

#### 3.5.2 Detailed Construction

Let  $A_G = (Q, \Sigma, Q_0, \delta)$  be a TA as an XML schema and let  $T_a = (Q_a, \Sigma, \Delta, \{q_a^0\}, \delta_a)$ and  $T_s = (Q_s, \Sigma, \Delta, \{q_s^0\}, \delta_s)$  be LDTT as authorized and unauthorized queries, respectively. Our algorithm works as follows:

**Step 1.** Construct the following two TAs  $A_a$  and  $A_s$  from  $T_a$  and  $T_s$ :

- $A_a = (\delta_a \cup {\{\perp\}}, \Sigma, \delta_{a0}, \delta'_a)$  where  $\perp \notin Q_a$ ,  $\delta_{a0}$  is the set of rules in  $\delta_a$  of which left hand side contains  $q_a^0$  and  $\delta_a'$  is the smallest set such that
	- for each  $\rho = q(\sigma(x_1, x_2)) \rightarrow C[q_1(x_1), q_2(x_2)] \in \delta_a$  with  $q, q_1, q_2 \in Q_a$ ,  $\sigma \in \Sigma$  and  $C \in \mathcal{C}(\Delta, \mathcal{X}_2)$ ,

$$
\rho \to \sigma(\tilde{q}_1, \tilde{q}_2) \in \delta_a'
$$

where for each  $i \in \{1,2\}$ , if the right hand side of  $\rho$  (abbreviated as  $RHS(\rho)$  contains  $x_i$  then  $\tilde{q}_i = \rho_i$  for any  $\rho_i \in \delta_a$  of which left hand side contains  $q_i$ ; otherwise  $\tilde{q}_i = \perp$ .

- for each  $\rho = q(\#) \to u$  where  $q \in Q_a$  and  $u \in \mathcal{T}_{\Delta}, \rho \to \# \in \delta'_a$ .
- for each  $\sigma \in \Sigma$ ,  $\bot \to \sigma(\bot, \bot) \in \delta_a'$ .
- $\bullet \ \bot \to \# \in \delta_a'.$

If the variable  $x_i$  disappears from RHS( $\rho$ ), we use the dummy state ⊥ instead of  $q_i$  so that  $A_a$  goes through the subtree substituted into  $x_i$  by the rule  $\bot \to \sigma(\bot, \bot).$ 

 $-A_s=(\delta_s\cup\{\perp\},\Sigma,\delta_{s0},\delta'_s)$  is defined from  $T_s$  in the same way as  $A_a$ . We can see that  $A_a$  and  $A_s$  constructed from  $T_a$  and  $T_s$  as above satisfy the three conditions listed in Section 3.5.1. Note that  $A_a$  and  $A_s$  are unambiguous TAs because  $T_a$  and  $T_s$  are top-down deterministic and thus it is uniquely determined which rule is applied to each node.

- **Step 2.** Construct the product TA  $A_I$  of  $A_a$ ,  $A_s$ , and  $A_G$ . More specifically, construct the following tree automaton  $A_I = (Q'_a \times Q'_s \times Q, \Sigma, Q_a^0 \times Q_s^0 \times Q_0, \delta_I)$ from  $A_a = (Q'_a, \Sigma, Q_a^0, \delta'_a), A_s = (Q'_s, \Sigma, Q_s^0, \delta'_s), \text{ and } A_G = (Q, \Sigma, Q_0, \delta).$  $(q_a, q_s, q) \to \sigma((q_a^1, q_s^1, q^1), (q_a^2, q_s^2, q^2)) \in \delta_I$  if and only if  $q_a \to \sigma(q_a^1, q_a^2) \in \delta'_a$ ,  $q_s \to \sigma(q_s^1, q_s^2) \in \delta'_s$ , and  $q \to \sigma(q^1, q^2) \in \delta$ . Note that  $L(A_G) = L(A_I)$  because  $Dom(T_a) \supseteq L(A_G)$  and  $Dom(T_s) \supseteq L(A_G)$ . Also for every  $t \in L(A_I)$ , there are unique runs  $r_a, r_s$  of  $A_a, A_s$  respectively, on t because  $A_a, A_s$  are unambiguous; r is a run of  $A_G$  on t if and only if r' is a run of  $A_I$  on t such that  $r'(v) = (r_a(v), r_s(v), r(v))$  for every node  $v \in V(t)$ .
- **Step 3.** Let  $Q_I = Q'_a \times Q'_s \times Q$  be the set of states of  $A_I$ . Compute two subsets  $\mathcal{Q}_a^D$  and  $\mathcal{Q}_s^D$  of  $Q_I$  as follows:

$$
Q_a^D = \{ (q_a, q_s, q) \in Q_I \mid q_a = \bot \text{ or } q_a = \rho
$$
  
where  $\rho$  is an erasing rule of  $T_a$ ,  

$$
Q_s^D = \{ (q_a, q_s, q) \in Q_I \mid q_s = \bot \text{ or } q_s = \rho
$$
  
where  $\rho$  is an erasing rule of  $T_s$ .

 $A_I$  assigns a state  $(q_a, q_s, q) \in \mathcal{Q}_a^D$  (rsp.  $\mathcal{Q}_s^D$ ) to a node v if and only if  $T_a$ (rsp.  $T_s$ ) deletes v by a subtree-deleting rule.

**Step 4.** Compute Q' consisting of  $q_I \in Q_I$  such that  $q_I$  is recursive through a nonempty path of the dependency graph  $G_{A_I}$  such that all the states in the path are in  $\mathcal{Q}_a^D$  and some state in the path is not in  $\mathcal{Q}_s^D$ . Let  $\mathcal{Q}_a^D$  =  $\{1, 2, ..., n\}$  where  $n = |Q_a^D|$ . We can compute Q' by dynamic programming using  $\mathcal{D}_k[i,j]$   $(k,i,j \in \mathcal{Q}_a^D)$ . We intend  $\mathcal{D}_k[i,j] = 0$  if there is no path from i to j via 1, 2, ..., k.  $\mathcal{D}_k[i, j] = 1$  if there is a path from i to j via 1, 2, ..., k and every node in such a path belongs to  $\mathcal{Q}_s^D$ .  $\mathcal{D}_k[i,j] \geq 2$  if there is a path from i to j via  $1, 2, ..., k$  such that at least one node in the path does not belong to  $\mathcal{Q}_s^D$ .

 $\mathcal{D}_0[i, j]$  can be defined as follows: for each  $i, j \in \mathcal{Q}_a^D$ 

- if  $\exists k \in \mathcal{Q}_{a}^{D} \exists \sigma \in \Sigma$ :  $i \to \sigma(j,k) \in \delta_I$  or  $i \to \sigma(k,j) \in \delta_I$  then - if  $i, j \in \mathcal{Q}_s^D$  then  $\mathcal{D}_0[i, j] := 1$ – else  $\mathcal{D}_0[i, j] := 2$
- else  $\mathcal{D}_0[i, j] := 0$ .

We compute  $\mathcal{D}_k[i, j]$   $(1 \leq k \leq n)$  as follows:  $\mathcal{D}_k[i, j] = \max\{\mathcal{D}_{k-1}[i, k] \times$  $\mathcal{D}_{k-1}[k,j], \mathcal{D}_{k-1}[i,j]$ . Then  $Q'$  can be obtained as  $\{i \mid \mathcal{D}_n[i,i] \geq 2\}$ .

- **Step 5.** Construct  $A_{inf}$  such that  $t \in L(A_{inf})$  if and only if there is a run  $r_t^I$ of  $A_I$  on t that assigns a state in  $Q'$  to at least one node of t as follows:  $A_{inf} = (Q_I \times \{0, 1\}, \Sigma, \{q_a^0\} \times \{q_s^0\} \times Q_0 \times \{1\}, \delta_I'')$  where  $\delta_I''$  is the smallest set such that
	- for each  $q \to \sigma(q_1, q_2) \in \delta_I$ , if  $q \in Q'$  then  $(q, 1) \to \sigma((q_1, i), (q_2, j)) \in$  $\delta''_I$  for each  $i, j \in \{0, 1\}$ ; otherwise,  $(q, i||j) \rightarrow \sigma((q_1, i), (q_2, j)) \in \delta''_I$  for each  $i, j \in \{0, 1\}$ , where  $\parallel$  is the boolean operator OR, and
	- for each  $q \to \# \in \delta_I$ , if  $q \in Q'$  then  $(q, 1) \to \# \in \delta_I''$ ; otherwise  $(q, 0) \rightarrow # \in \delta_I''$ .

The last component  $\{0,1\}$  of each state is for checking whether at least one state in  $Q'$  appears in a run.

- **Step 6.** Construct  $A_{fin}$  such that  $L(A_{fin}) = L(A_I) L(A_{inf})$ . More concretely, construct a complement TA  $A_{inf}^c$  of  $A_{inf}$  and then construct an intersection TA of  $A_I$  and  $A_{inf}^c$ . Note that TAs are closed under Boolean operations [5].
- Step 7. Decide whether  $T_a(L(A_{fin})) \subseteq T_a(L(A_{inf}))$ . If  $T_a(L(A_{fin})) \subseteq T_a(L(A_{inf}))$ then output "schema ∞-secret"; Otherwise, output "not schema ∞-secret."

Note that any LDTT T preserves regularity, that is, for any regular tree language  $L, T(L)$  is also a regular language, and a TA which recognizes  $T(L)$ can be constructed. Also, the inclusion problem is decidable for regular tree languages.

**Example 3.5.1.** Using the above algorithm, we can decide  $\infty$ -secrecy for Example 3.3.3 as follows:

$T_a^i$ $\tau$ $\perp$ <sub>s</sub>	$L(A_{inf})$	$L(A_{fin})$	$T_a^i(L(A_{inf}))$	$T_a^i(L(A_{fin}))$	
$T_{a}^{1}, T_{s}^{1}$	$L(A_G)$		$\it a$		yes
$T_{a}^{2}, T_{s}^{1}$	$L(A_G)$		$a \cup \tilde{a}$		yes
$T_{a}^{1}, T_{s}^{2}$	$\tilde{a}s^+$	$as^+$	$\boldsymbol{a}$	$\it a$	yes
$T_a^2, T_s^2$	$\tilde{a}s^+$	$as^+$	$\tilde{\phantom{a}}$ $\mathfrak a$	$\it a$	no

Table 3.2. The execution results of the algorithm for Example 3.3.3.

We let  $H = T_a^i(L(A_{fin})) \subseteq T_a^i(L(A_{inf})).$ 

Consider the third case  $(A_G, T_a^1, T_s^2)$  in Table 3.2. We assign a label to each rule of  $T_a^1$  and  $T_s^2$  as  $\delta_a^1 = \{ \rho_a^1: q(a(x_1, x_2)) \to a(q(x_1), q(x_2)), \rho_a^2: q(\tilde{a}(x_1, x_2)) \to a(q(x_1), q(x_2)) \}$  $a(q(x_1), q(x_2)), \rho_a^3:q(s(x_1, x_2)) \rightarrow q(x_1), \rho_a^4:q(\#) \rightarrow \#$ }.  $\delta_s^2 = \{\rho_s^1:q(a(x_1, x_2)) \rightarrow$  $a(\textbf{\#}, \textbf{\#}), \rho_s^2:q(\tilde{a}(x_1, x_2)) \rightarrow \tilde{a}(q(x_1), q(x_2)), \rho_s^3:q(s(x_1, x_2)) \rightarrow s(q(x_1), q(x_2)),$  $\rho_s^4:q(\#) \to \#$ . We can construct  $A_a^1, A_s^2, A_I, A_{inf}$  and obtain  $Q'$  as follows. Note that useless states and rules are removed at (Step 2) and (Step 5).

• (Step 1)  $A_a^1 = (Q_a^1)$  $\prime, \Sigma, \delta^1_a, \delta^1_a$ ') where  $Q_a^{1'} = \delta_a^1 \cup {\{\perp\}}, \delta_a^{1'} = {\{\rho_a^1 \to a(\rho_a^i, \rho_a^j),\}$  $\rho_a^2 \rightarrow \tilde{a}(\rho_a^i, \rho_a^j), \rho_a^3 \rightarrow s(\rho_a^i, \perp), \rho_a^4 \rightarrow \#$ ,  $\perp \rightarrow \sigma(\perp, \perp), \perp \rightarrow \#$ } with  $1 \leq i \leq 4, 1 \leq j \leq 4$  and  $\sigma \in \{a, \tilde{a}, s\}.$   $A_s^2 = (Q_s^2)$  $\mathcal{L}, \mathcal{S}_s^2, \mathcal{S}_s^2$  $\prime$ ) where  $Q_s^{2'} = \delta_s^2 \cup {\{\perp\}}, \ \delta_s^{2'} = \{\rho_s^1 \to a(\perp,\perp), \ \rho_s^2 \to \tilde{a}(\rho_s^i, \rho_s^j), \ \rho_s^3 \to s(\rho_s^i, \rho_s^j),$  $\rho_s^4 \rightarrow #, \perp \rightarrow \sigma(\perp, \perp), \perp \rightarrow #$ } with  $1 \leq i \leq 4, 1 \leq j \leq 4$  and  $\sigma \in \{a, \tilde{a}, s\}.$ 

• (Step 2) 
$$
A_I = (Q_a^{1'} \times Q_s^{2'} \times Q, \Sigma, \{(\rho_a^1, \rho_s^1, q_0), (\rho_a^2, \rho_s^2, q_0)\},
$$
  
\n $\delta_I$ ) where  $\delta_I = \{(\rho_a^1, \rho_s^1, q_0) \to a((\rho_a^3, \bot, q_1), (\rho_a^4, \bot, q_{\#})),$   
\n $(\rho_a^2, \rho_s^2, q_0) \to \tilde{a}((\rho_a^3, \rho_s^3, q_1), (\rho_a^4, \rho_s^4, q_{\#})),$   
\n $(\rho_a^3, \rho_s^3, q_1) \to s((\rho_a^3, \rho_s^3, q_1), (\bot, \rho_s^4, q_{\#})),$   
\n $(\rho_a^3, \rho_s^3, q_1) \to s((\rho_a^4, \rho_s^4, q_{\#}), (\bot, \rho_s^4, q_{\#})),$   
\n $(\rho_a^3, \bot, q_1) \to s((\rho_a^3, \bot, q_1), (\bot, \bot, q_{\#})),$   
\n $(\rho_a^3, \bot, q_1) \to s((\rho_a^4, \bot, q_{\#}), (\bot, \bot, q_{\#})),$   
\n $(\rho_a^4, \rho_s^4, q_{\#}) \to \#, (\rho_a^4, \bot, q_{\#}) \to \#, (\bot, \rho_s^4, q_{\#}) \to \#, (\bot, \bot, q_{\#}) \to \#.$ 

- (Step 3)  $\mathcal{Q}_a^D$  is the subset of states of which first component is  $\rho_a^3$  or  $\perp$ .  $\mathcal{Q}_s^D$ is the subset of states of which second component is  $\bot$ .
- (Step 4) The dependency graph  $G_{A_I}$  of  $A_I$  is shown in Figure 3.4.  $Q' =$  $\{(\rho_a^3, \rho_s^3, q_1)\}.$
- (Step 5)  $A_{inf} = (Q_I \times \{0, 1\}, \Sigma, \{((\rho_a^2, \rho_s^2, q_0), 1)\}, \delta_I'')$  where  $\delta_I'' = \{((\rho_a^2, \rho_s^2, q_0), 1) \rightarrow$  $\tilde{a}(((\rho_a^3, \rho_s^3, q_1), 1), ((\rho_a^4,$  $(\rho_s^4, q_\#), 0)), ((\rho_a^3, \rho_s^3, q_1), 1) \rightarrow s(((\rho_a^3, \rho_s^3, q_1), 1), ((\perp, \rho_s^4,$  $(q_{\#}),0)), ((\rho_a^3, \rho_s^3, q_1), 1) \rightarrow s(((\rho_a^4, \rho_s^4, q_{\#}), 0), ((\bot, \rho_s^4, q_{\#}),$ 0)),  $((\rho_a^4, \rho_s^4, q_\#), 0) \to \#$ ,  $((\rho_a^4, \bot, q_\#), 0) \to \#$ ,  $((\bot, \rho_s^4, q_\#), 0) \to \#$ .
- (Step 6)  $A_{fin}$  is a TA such that  $L(A_{fin}) = L(A_I) L(A_{inf}).$
- (Step 7) We have  $L(A_{inf}) = \tilde{a}s^+$ ,  $L(A_{fin}) = L(A_G) L(A_{inf}) = as^+$ .  $T_a^1(L(A_{fin})) \subseteq T_a^1(L(A_{inf}))$  holds and  $(A_G, T_a^1, T_s^2)$  is schema  $\infty$ -secret because  $T_a^1(L(A_{fin})) = a$  and  $T_a^1(L(A_{inf})) = a$ . See Table 3.2.



Figure 3.4. The dependency graph  $G_{A_I}$ .

#### 3.5.3 Correctness

We give several lemmas for the proof of the correctness.

**Lemma 3.5.2.** Let t be an arbitrary tree in  $L(A_I)$ . For any node  $v_1$  and any descendant  $v_2$  of  $v_1$  in t such that

- there is a run  $r_t^I$  of  $A_I$  on t such that  $r_t^I(v_1) = r_t^I(v_2)$  and
- every node v that appears in the path from  $v_1$  to  $v_2$  (inclusive) is mapped to a state in  $\mathcal{Q}_a^D$  (rsp. in  $\mathcal{Q}_s^D$ ) by the run  $r_t^I$ ,

 $T_a(t) = T_a(t[v_1 \leftarrow t/v_2])$  (rsp.  $T_s(t) = T_s(t[v_1 \leftarrow t/v_2]))$  (see Figure 3.5).



Figure 3.5. Lemma 3.5.2.

*Proof.* Let t be an arbitrary tree in  $L(A_I)$  and let v be an arbitrary node in t. Let  $r_t^I$  be an arbitrary run of  $A_I$  on t. By definition of  $A_a$ ,  $A_s$ , and  $A_I$ , we have the following two facts:

- If  $r_t^I(v) = (\perp, q_s, q) \in \mathcal{Q}_a^D$  then for every descendant v' of v,  $r_t^I(v') =$  $(\perp, q'_s, q') \in \mathcal{Q}_a^D$  for some  $q'_s \in Q_s$  and  $q' \in Q$ .
- If  $r_t^I(v) = (q_a, q_s, q) \in \mathcal{Q}_a^D$  where  $q_a \neq \bot$  then for either one node v' of the two children of  $v, r_t^I(v') = (\perp, q_s', q') \in \mathcal{Q}_a^D$  for some  $q_s' \in Q_s$  and  $q' \in Q$ .



Figure 3.6. Lemma 3.5.3.

Let  $v_1$  be an arbitrary node in t, and let  $v_2$  be an arbitrary descendant of  $v_1$ . We assume that  $r_t^I(v_1) = r_t^I(v_2) \in \mathcal{Q}_a^D$  and that  $r_t^I(v) \in \mathcal{Q}_a^D$  for each v that appears in the path from  $v_1$  to  $v_2$ . By the facts we stated above, for each node v that is a descendant of  $v_1$  but not a descendant of  $v_2$ ,  $r_t^I(v) \in \mathcal{Q}_a^D$ . Since nodes mapped to states in  $\mathcal{Q}_a^D$  by  $r_t^I$  are deleted by  $T_a$ , it holds that  $T_a(t) = T_a(t[v_1 \leftarrow t/v_2]).$ 

The case for  $T_s$  and  $\mathcal{Q}_s^D$  can be shown in the same way.

 $\Box$ 

**Lemma 3.5.3.** For every  $\tau \in L(A_G)$ , if  $T_a(\tau) \in T_a(L(A_{inf}))$  then  $T_s(T_a^{-1}(T_a(\tau)) \cap$  $L(A_G)$ ) is infinite.

*Proof.* Assume that  $T_a(\tau) \in T_a(L(A_{inf}))$ , then there is  $t \in L(A_{inf})$  such that  $T_a(t) = T_a(\tau)$  (see Figure 3.6). Note that if  $\tau \in L(A_{inf})$  we just let  $t = \tau$ . By the definition of  $A_{inf}$ , there is a run  $r_t^I$  of  $A_I$  on t that assigns a state in  $Q'$  to at least one node of t. Let v be an arbitrary node such that  $r_t^I(v) \in Q'$ . By the definition of Q', there are another tree  $t' \in L(A_{inf})$  having a node  $v_1$  and its proper descendant  $v_2$ , and a run  $r_{t'}^I$  of  $A_I$  on  $t'$  such that

- $t'[v_1 \leftarrow t'/v_2] = t$  and  $t/v = t'/v_2$ ,
- $r_{t'}^I(v_1) = r_{t'}^I(v_2) = r_t^I(v) \in Q'$ , and
- all the states in the path from  $v_1$  and  $v_2$  are in  $\mathcal{Q}_a^D$  and some state in the path is not in  $\mathcal{Q}_s^D$ .



Figure 3.7. Lemma 3.5.4.

By Lemma 3.5.2,  $T_a(t') = T_a(t)$ . On the other hand,  $T_s(t') \neq T_s(t)$  because there is some state  $q_s \notin \mathcal{Q}_s^D$  between  $v_1$  and  $v_2$  in  $t'$  and thus at least one more node labeled with an output symbol in  $\Delta$  is emitted by  $T_s$ , that is,  $|T_s(t')| \geq |T_s(t)| + 1$ . Let  $t^k$  and  $t^{k+1}$  be the trees in  $L(A_{inf})$  obtained from t by pumping at v as above k and  $k + 1$  times, respectively, then  $T_a(t^k) = T_a(t^{k+1})$  and  $T_s(t^k) \neq T_s(t^{k+1})$ . Thus,  $T_s(T_a^{-1}(T_a(t)) \cap L(A_G))$  is infinite.  $\Box$ 

**Lemma 3.5.4.** For every  $t \in L(A_{fin})$ , if  $T_a(t) \notin T_a(L(A_{inf}))$  then  $T_s(T_a^{-1}(T_a(t)) \cap$  $L(A_G)$ ) is finite (see Figure 3.7).

*Proof.* Let t be an arbitrary tree in  $L(A_{fin})$  and let  $r_t^I$  be an arbitrary run of  $A_I$ on t. For any node  $v_1$  and any descendant  $v_2$  of  $v_1$  in t such that

- $r_t^I(v_1) = r_t^I(v_2)$  and
- every node v that appears in the path from  $v_1$  to  $v_2$  is mapped to a state in  $\mathcal{Q}_a^D$  by  $r_t^I$ ,

it holds not only that  $T_a(t) = T_a(t[v_1 \leftarrow t/v_2])$  but also that  $T_s(t) = T_s(t[v_1 \leftarrow t/v_2])$  $t/v_2$ ]). The reason is as follows: let  $\bar{q} = r_t^I(v_1) = r_t^I(v_2)$ , then  $\bar{q}$  is recursive through a path in t such that all of the states in the path are in  $\mathcal{Q}_a^D$ . By the fact that  $L(A_{fin}) = L(A_I) - L(A_{inf})$  and the construction of  $A_{inf}$ , there is no run of  $A_I$  on t that assigns a state in  $Q'$  to some node of t (see Step 5 in Section 3.5.2). Hence, we must have that  $r_t^I(v_1) = r_t^I(v_2) \in \mathcal{Q}_s^D$  and  $r_t^I(v) \in \mathcal{Q}_s^D$  for each v which appears in the path from  $v_1$  to  $v_2$ . Thus, by Lemma 3.5.2,  $T_s(t) = T_s(t[v_1 \leftarrow t/v_2]).$ 

Now consider an arbitrary tree  $t \in L(A_{fin})$  such that  $T_a(t) \notin T_a(L(A_{inf}))$ , and let  $t_0 = T_a(t)$ . For any  $t' \in T_a^{-1}(t_0)$  and any run  $r_{t'}^I$  of  $A_I$  on  $t'$ ,  $|\{v \in I\}|$  $V(t') | r_t^I(v) \notin \mathcal{Q}_a^D$  | ≤ |t<sub>0</sub>| because a node labeled with an output symbol in  $\Delta$ is emitted every time a rule that contains  $q \notin \mathcal{Q}_a^D$  in its left-hand side is applied. Thus, in any path of t', there are at most  $|t_0|$  nodes to which  $A_I$  assigns states not in  $\mathcal{Q}_a^D$ . Let l be the number of such nodes. In addition, in the path, there are  $l + 1$  intervals separated by the l nodes, and  $A_I$  assigns states in  $\mathcal{Q}_a^D$  to all nodes in the intervals. Hence, using the fact we have stated in the beginning of this proof, we can say that for each  $t' \in T_a^{-1}(t_0)$ , there is a tree  $t''$  such that  $T_s(t'') = T_s(t')$  and the height of t'' is at most  $|t_0| + (|t_0| + 1)|\mathcal{Q}_a^D|$ . Thus, there is a subset  $L_f$  of the set of all trees of height at most  $|t_0| + (|t_0| + 1)|\mathcal{Q}_a^D|$  such that  $T_s(L_f) = T_s(T_a^{-1}(t_0))$ . Since  $T_s$  is functional and  $L_f$  is finite,  $T_s(T_a^{-1}(t_0) \cap L(A_G))$ is finite.  $\Box$ 

We now give a lemma for correctness of our algorithm.

**Lemma 3.5.5.**  $T_a(L(A_{fin})) \subseteq T_a(L(A_{inf}))$  if and only if  $T_s(T_a^{-1}(T_a(t)) \cap L(A_G))$ is infinite for every  $t \in L(A_G)$ .

*Proof.* Assume that  $T_a(L(A_{fin})) \subseteq T_a(L(A_{inf}))$ . We have  $T_a(t) \in T_a(L(A_{inf}))$  for every  $t \in L(A_G)$ . By Lemma 3.5.3,  $T_s(T_a^{-1}(T_a(t)) \cap L(A_G))$  is infinite for every  $t \in L(A_G)$ .

On the other hand, assume that  $T_a(L(A_{fin}))\mathcal{Q}T_a(L(A_{inf}))$ , and then there is some  $t \in L(A_{fin}) \subseteq L(A_G)$  such that  $T_a(t) \notin T_a(L(A_{inf}))$ . Thus, by Lemma 3.5.4,  $T_s(T_a^{-1}(T_a(t)) \cap L(A_G))$  is finite.  $\Box$ 

By lemma 3.5.5, we obtain the following theorem.

**Theorem 3.5.6.**  $\infty$ -SS is decidable for LDTT.

#### 3.5.4 Time Complexity

Here, we show that  $\infty$ -SS for LDTT is EXPTIME-complete. First we show the time complexity of the proposed algorithm for ∞-SS where queries are represented by LDTT is in EXPTIME. Next we prove that  $\infty$ -SS for LDTT is EXPTIMEhard.

We estimate the time and space complexities of the proposed algorithm.

- **Step 1.** Construct two TAs  $A_a$  and  $A_s$  from  $T_a$  and  $T_s$ : it takes  $O(|T_a|^3)$  (rsp.  $O(|T_s|^3)$ ) time to construct  $A_a$  (rsp.  $A_s$ ), and its size is  $O(|T_a|^3)$  (rsp.  $O(|T_s|^3)).$
- **Step 2.** Construct the product TA  $A_I$  from  $A_s$ ,  $A_a$  and  $A_G$ : it takes  $O(|A_a| \times$  $|A_s| \times |A_G|$ ) time, and its size is  $O(|A_a| \times |A_s| \times |A_G|)$ .
- **Step 3.** Compute  $\mathcal{Q}_a^D$  and  $\mathcal{Q}_s^D$ : it takes  $O(|A_I| \times |A_a|)$  (rsp.  $O(|A_I| \times |A_s|)$ ) time to construct  $\mathcal{Q}_a^D$  (rsp.  $\mathcal{Q}_s^D$ ), and the sizes of  $\mathcal{Q}_a^D$  and  $\mathcal{Q}_s^D$  are  $O(|A_I|)$ .
- **Step 4.** Construct  $Q'$  from  $A_I$ ,  $Q_a^D$ , and  $Q_s^D$ : construction of  $Q'$  takes  $O(|A_I|^3)$ time, and its size is  $O(|A_I|)$  because  $Q'$  is a subset of  $Q_I$ .
- **Step 5.** Construct  $A_{inf}$  from  $A_I$  and  $Q'$ : construction of  $A_{inf}$  takes  $O(|A_I| \times |Q'|)$ time, and  $|A_{inf}|$  is  $O(|A_I|)$ .
- Step 6. Construct  $A_{fin}$  such that  $L(A_{fin}) = L(A_I) L(A_{inf})$ : construction of a complement TA  $A_{inf}^c$  of  $A_{inf}$  takes exponential time and its size is exponential in  $|A_{inf}|$  at the worst. Construction of an intersection TA  $A_{fin}$ of  $A_I$  and  $A_{inf}^c$  takes  $O(|A_I| \times |A_{inf}^c|)$  time and its size is  $O(|A_I| \times |A_{inf}^c|)$ .
- **Step 7.** Decide whether  $T_a(L(A_{fin})) \subseteq T_a(L(A_{inf}))$ : a TA  $B_{fin}$  recognizing  $T_a(L(A_{fin}))$  can be constructed in  $O(|T_a| \times |A_{fin}|)$  time. Similarly, a TA  $B_{inf}$  recognizing  $T_a(L(A_{inf}))$  can be constructed in  $O(|T_a| \times |A_{inf}|)$  time. Note that the size of  $B_{fin}$  is exponential in the sizes of  $T_a, T_s$ , and  $A_G$  but that of  $B_{inf}$  is polynomial. Thus, we can obtain a complement TA  $B_{inf}^c$ of  $B_{inf}$  of exponential size, and deciding whether  $L(B_{fin}) \cap L(B_{inf}^c) = \emptyset$  is possible in  $O(|B_{fin}| \times |B_{inf}^c|)$  time.

Therefore, the total time complexity of our algorithm is in EXPTIME.

#### Theorem 3.5.7.  $\infty$ -SS is EXPTIME-complete for LDTT.

*Proof.* We showed  $\infty$ -SS is in EXPTIME solvable and thus we show  $\infty$ -SS is EXPTIME-hard by reduction from inclusion of regular tree languages, which is known to be EXPTIME-complete. The reduction uses Lemma 3.5.5.

Consider any two TAs  $A_1 = (Q_1, \Sigma, Q_{10}, \delta_1)$  and  $A_2 = (Q_2, \Sigma, Q_{20}, \delta_2)$ . We construct  $A_G$ ,  $T_a$ , and  $T_s$  such that  $T_a(L(A_{fin})) = L(A_1)$  and  $T_a(L(A_{inf})) =$ 

 $L(A_2)$ . Without loss of generality, neither  $A_1$  nor  $A_2$  accepts the tree having only #, and  $Q_1$  and  $Q_2$  are disjoint. Let  $\Sigma_{A_i} = \{a_{(q,q_1,q_2)} | q \to a(q_1,q_2) \in \delta_i\}$   $(i = 1, 2)$ . For  $a_{(q,q_1,q_2)} \in \Sigma_{A_1} \cup \Sigma_{A_2}$ , let  $\pi(a_{(q,q_1,q_2)}) = a$ . Note that  $\Sigma_{A_1}$  and  $\Sigma_{A_2}$  are disjoint because so are  $Q_1$  and  $Q_2$ . We construct the following TA  $A_G$  and transducers  $T_a$  and  $T_s$ :

- $A_G = (Q_1 \cup Q_2 \cup \{q_0\}, \Sigma_{A_1} \cup \Sigma_{A_2} \cup \{s\}, Q_{10} \cup \{q_0\}, \delta_G)$  where  $q_0$  is a new state, s is a new symbol, and  $\delta_G$  is the smallest set such that
	- for each  $a \in \Sigma$ , if  $q \to a(q_1, q_2) \in \delta_1 \cup \delta_2$  then  $q \to a_{(q,q_1,q_2)}(q_1, q_2) \in \delta_G$ ,
	- for each  $q_{20} \in Q_{20}$ ,  $q_0 \to s(q_{20}, q_{\#}) \in \delta_G$ ,
	- $q_0 \rightarrow s(q_0, q_\#) \in \delta_G$ , and
	- $q_{\#} \rightarrow \# \in \delta_G.$
- $\bullet$   $T_a = (\{q_a\}, \Sigma_{A_1} \cup \Sigma_{A_2} \cup \{s\}, \Sigma, \{q_a\}, \delta_a)$  where  $\delta_a = \{q_a(\tilde{a}(x_1, x_2)) \rightarrow$  $a(q_a(x_1), q_a(x_2)) | \tilde{a} \in \Sigma_{A_1} \cup \Sigma_{A_2}, \pi(\tilde{a}) = a \} \cup \{q_a(s(x_1, x_2)) \rightarrow q_a(x_1), q_a(\#) \rightarrow$ #}.
- $T_s = (\{q_s\}, \Sigma_{A_1} \cup \Sigma_{A_2} \cup \{s\}, \Sigma_{A_1} \cup \Sigma_{A_2} \cup \{s\}, \{q_s\}, \delta_s)$  where  $\delta_s = \{q_s(\sigma(x_1, x_2)) \rightarrow$  $\sigma(q_s(x_1), q_s(x_2)) \mid \sigma \in \Sigma_{A_1} \cup \Sigma_{A_2} \cup \{s\} \cup \{q_s(\#) \to \# \},$  which represents the identity transformation on trees over  $\Sigma_{A_1} \cup \Sigma_{A_2} \cup \{s\}.$

Note that  $A_G$  is an unambiguous TA, and  $T_a$  and  $T_s$  are LDTTs.  $L(A_G)$  consists of two disjoint sets: the set  $L_1$  of trees over  $\Sigma_{A_1}$  corresponding to trees in  $L(A_1)$  and the set  $L_2$  of trees over  $\Sigma_{A_2} \cup \{s\}$  corresponding to trees in  $L(A_2)$ . For  $t \in L(A_1)$ , the size of  $T_a^{-1}(t) \cap L_1$  is at most the number of runs of  $A_1$  on t. On the other hand, for  $t \in L(A_2)$ , there are infinitely many trees in  $L_2$  that are transformed to t by  $T_a$  because trees in  $L_2$  are allowed to have any number of s on the top and  $T_a$ deletes all the ss. From the above facts, we can see that  $A_{fin}$  and  $A_{inf}$  constructed by our decision algorithm in Section 3.5.2 satisfy that  $T_a(L(A_{fin})) = L(A_1)$  and  $T_a(L(A_{inf})) = L(A_2)$ . Hence, from Lemma 4,  $L(A_1) \subseteq L(A_2)$  if and only if  $(A_G, T_a, T_s)$  is  $\infty$ -secret. Thus, we obtain EXPTIME-hardness of  $\infty$ -SS.  $\Box$ 

EXPTIME-hardness of LDBT can be shown in a similar way as above.

#### 3.5.5 Extension to  $LDTT^R$

We show that  $\infty$ -SS is decidable for LDTT<sup>R</sup>s by revising Steps 1 and 3 of the algorithm in Section 3.5.2.

**Step 1.** Construct the following TA  $A_a$  from a given LDTT<sup>R</sup>  $T_a = (Q_a, \Sigma, \Delta, q_a^0, \delta_a)$ : **Step 1.1** Let  $\mathcal A$  be the set of all the TAs occurring as ranges (look-ahead) of variables in the rules of  $T_a$ . Let  $A_{\rho,i}$  denote the TA of ranges of variable  $x_i$  in the rule  $\rho$ . For each  $A_{\rho,i} \in \mathcal{A}$ , construct a bottom-up deterministic [5] and complete TA  $A'_{\rho,i}$  equivalent to  $A_{\rho,i}$ . Let  $A'_{\rho,i} = (Q'_{\rho,i}, \Sigma, Q'_{0,\rho,i}, \delta_{\rho,i})$ , and let  $\mathcal{A}' = \{A'_{\rho,i} \mid A_{\rho,i} \in \mathcal{A}\}.$ 

**Step 1.2** Construct a product TA  $A^{rl} = (Q^{rl}, \Sigma, Q_0^{rl}, \delta^{rl})$  of all the TAs  $A'_{\rho,i}$ . For  $\bar{q} \in Q^{rl}$ , let  $\pi_{\rho,i}(\bar{q})$  be the state of  $A'_{\rho,i}$  occurring in  $\bar{q}$ .

**Step 1.3** Construct  $A_a = ((\delta_a \cup \{\perp\}) \times Q^{rl}, \Sigma, \delta_{a0} \times Q^{rl}, \delta')$  where  $\delta_{a0}$  is the set of rules in  $\delta_a$  of which left hand side contains  $q_a^0$  and  $\delta'$  is the smallest set such that

• for each  $\rho = q(\sigma(x_1 : A_1, x_2 : A_2)) \rightarrow C[q_1(x_1), q_2(x_2)] \in \delta_a$  where  $q_1, q_2, q \in Q_a$  and  $\sigma \in \Sigma$ ,

$$
(\rho, \bar{q}) \to \sigma((\tilde{q}_1, \bar{q}_1), (\tilde{q}_2, \bar{q}_2)) \in \delta'
$$

where  $\bar{q} \to \sigma(\bar{q}_1, \bar{q}_2) \in \delta^{rl}$ , for  $i \in \{1, 2\}, \pi_{\rho,i}(\bar{q}_i) \in Q'_{0,\rho,i}$ , and

- if RHS( $\rho$ ) contains  $x_i$  then  $\tilde{q}_i = \rho_i$  for any  $\rho_i \in \delta_a$  of which left hand side contains  $q_i$ , and
- if RHS( $\rho$ ) does not contain  $x_i$ , then  $\tilde{q}_i = \perp$ .
- for each  $r = q(\#) \to u \in \delta_a$ ,  $(\rho, \bar{q}) \to \# \in \delta'$  where  $\bar{q} \to \# \in \delta^{\rho l}$ .
- for each  $\bar{q} \to \sigma(\bar{q}_1, \bar{q}_2) \in \delta^{rl}$ ,  $(\perp, \bar{q}) \to \sigma((\perp, \bar{q}_1), (\perp, \bar{q}_2)) \in \delta'.$
- for each  $\bar{q} \to \# \in \delta^{rl}, (\perp, \bar{q}) \to \# \in \delta'.$

 $A_s = (\delta_s \cup {\{\perp\}} \times Q^{rl}, \Sigma, \delta_{s0} \times Q^{rl}, \delta')$  can be constructed from  $T_s$  in the same way as  $A_a$ .

**Step 3.** Let  $Q_I$  be the set of states of  $A_I$ . Compute two subsets  $\mathcal{Q}_a^D$  and  $\mathcal{Q}_s^D$  of  $Q_I$  as follows:

It is also known that  $LDTT^R$  preserves regularity [7]. In the same way as Section 3.5.3, we can show that the revised algorithm for  $LDTT^{R}$ s works correctly. The sizes of  $A_a$  and  $A_s$  are exponential in  $|T_a|$  and  $|T_s|$ , respectively, because  $|A^{rl}|$ is exponential in |A| and |A| is proportional to the product of  $|A^{rl}|$ , |Q|, and  $|\delta|$ . Thus, the time complexity of the algorithm is doubly exponential.

**Theorem 3.5.8.**  $\infty$ -SS is decidable in 2-EXPTIME for LDTT<sup>R</sup>.

## 3.6. Discussion

#### 3.6.1 Multiple Authorized Queries

We have discussed  $\infty$ -SS for a single authorized query. Generalizing the results to the case that there are multiple authorized queries is important because using multiple authorized queries attackers can narrow down the candidates of secret information to less than using them independently in general. However,  $\infty$ -SS for more than one authorized queries is much complicated and might be undecidable at worst. In fact, the approach in Section 3.5.1 does not work well. Now we assume that LDTTs  $T_{a1}$  and  $T_{a2}$  are given as authorized queries, and  $T_s$  and  $A_G$ are given as an unauthorized query and a schema. Then, the following procedure which is almost the same as Section 3.5.2 is likely to work:

- **Step 1.** Construct  $A_{a1}$ ,  $A_{a2}$  and  $A_s$  from  $T_{a1}$ ,  $T_{a2}$ , and  $T_s$ , respectively.
- **Step 2.** Compute the product TA of  $A_{a1}$ ,  $A_{a2}$ ,  $A_s$  and  $A_G$ .
- **Step 3.** Compute  $\mathcal{Q}_{a1}^D$ ,  $\mathcal{Q}_{a2}^D$  and  $\mathcal{Q}_{s}^D$ , respectively.
- **Step 4.** Compute  $Q' = \{q_I \in Q_I | q_I \text{ is recursive through a nonempty path such }\}$ that all the states in the path are in  $\mathcal{Q}_{a1}^D \cap \mathcal{Q}_{a2}^D$  and some state in the path is not in  $\mathcal{Q}_s^D$ .}

Steps 5–6. These steps are the same as Section 3.5.2.

**Step 7.** Let  $(T_{a1}, T_{a2})(L(A_{fin})) = \{(T_{a1}(t), T_{a2}(t)) | t \in L(A_{fin})\}$  and  $(T_{a1}, T_{a2})$  $(L(A_{inf})) = \{(T_{a1}(t), T_{a2}(t)) \mid t \in L(A_{fin})\}.$  Then, decide whether  $(T_{a1}, T_{a2})$  $(L(A_{fin})) \subseteq (T_{a1}, T_{a2})(L(A_{inf})).$ 

Unfortunately, the above procedure does not work in Step 7 because we do not know whether  $(T_{a1}, T_{a2})(L(A_{fin})) \subseteq (T_{a1}, T_{a2})(L(A_{inf}))$  is decidable. At least,  $(T_{a1}, T_{a2})(L(A_{inf}))$  cannot be captured by any regular tree language in general.

Similarly, even for a single authorized query, when the authorized query  $T_a$ is given by a non-linear top-down tree transducer, which allows to copy subtrees twice or more,  $T_a(L)$  is not guaranteed to be regular for a regular tree language L. Thus, our algorithm cannot be extended directly to such cases.

#### 3.6.2 k-Secrecy Preservation through Updates

We would like to extend  $k$ -SS for deciding the k-secrecy preservation through updates. For some schema,  $k$ -secrecy is too strong to satisfy. On the other hand, a database instance is frequently updated in the real world. Consider the property that for any instance  $t$  which is  $k$ -secret, every instance obtained from t through updates is also k-secret. We call this property k-secrecy preserving property through updates, abbreviated as  $k$ -SP. Formally, given an XML schema  $A_G$ , an authorized query  $T_a$ , an unauthorized query  $T_s$ , and update operations U, we say that U preserves k-secrecy with respect to  $(A_G, T_a, T_s)$  if  $U(t)$  is k-secret whenever  $t \in L(A_G)$  is k-secret with respect to  $(A_G, T_a, T_s)$ . If a given schema, queries and update operations satisfy k-SP, then it suffices to guarantee that the initial instance is  $k$ -secret in order to keep every instance through updates k-secret.

We can show that a variant of  $\infty$ -SP is decidable by using the decidability result of  $\infty$ -SS. Consider a regular subset  $L_{sub}$  of  $L(A_G)$  such that every instance in  $L_{sub}$  is  $\infty$ -secret. Here, for update operations, we adopt UFO<sub>reg</sub> [22], which is a restricted model for XML update primitives of the W3C XQuery Update Facility. The variant of  $\infty$ -SP problem is to decide whether all possible instances obtained by updating instances in  $L_{sub}$  by applying any finite number of update operations are  $k$ -secret or not. Since it is known that  $\mathsf{UFO}_{\mathsf{reg}}$  has the forward regularity preserving property, the set  $post*(L_{sub})$  of the possible instances after

updates is also regular. Thus, to decide the variant of  $\infty$ -SP, we just decide whether  $(post^*(L_{sub}) \cap L(A_G), T_a, T_s)$  is k-secret or not.

# 3.7. Conclusion

In this chapter, we have discussed the problem of deciding whether every database instance conforming to a given XML schema is safe against inference attacks for given authorized and unauthorized queries in the sense of  $k$ -secrecy  $(k$ -SS).

We have shown that k-SS is undecidable for any natural number  $k > 1$  by reduction from PCP even when a query is given by a deterministic rational transducer on words (DRTW). For a positive result, we have shown that  $\infty$ -SS is EXPTIME-complete for LDTT and in 2-EXPTIME for  $LDTT<sup>R</sup>$ . The proposed method can be applied to linear deterministic bottom-up tree transducer and a similar result has been obtained in [17].

# Chapter 4

# Query-Based  $\ell$ -Diversity on Relational Databases

This chapter introduces a new privacy notion against inference attacks called query-based  $\ell$ -diversity by extending  $\ell$ -diversity [15] in relational databases with access control for queries. The basic notions of relational database is provided. Then we end with the experimental results conducted on two verification tools implemented by SQL and #SAT solver.

## 4.1. Introduction

As described in Chapter 2, there are well-known related notions such as kanonymity and  $\ell$ -diversity. Intuitively, a database is k-anonymous if for every individual  $x$ , there are at least  $k$  different records (or tuples in the relational database setting) which cannot be distinguished from the real record for x. A database is  $\ell$ -diverse if for every individual x, there are at least  $\ell$  different values of the sensitive information contained in the records which cannot be distinguished from the real record for x. Also, various methods are reported for transforming a given database into a database satisfying  $k$ -anonymity or  $\ell$ -diversity. However, these notions do not take the effect of access control for queries into consideration.

The goal of this study is to introduce a notion of the security against inference attack by extending  $\ell$ -diversity in relational databases with access control for queries. More specifically, we propose a new privacy notion called *query-based*  $\ell$ -

diversity. A database instance T is (query-based)  $\ell$ -diverse with respect to given authorized queries if an attacker cannot narrow down the number of possible values of the sensitive information for any individual to less than  $\ell$  by inference based on the result of the authorized queries on the instance  $T$  and the queries themselves. We provide two approaches to deciding the query-based  $\ell$ -diversity. In the first approach, a decision algorithm is given by using relational operations, which can be directly implemented by a relational database management system, e.g., SQL. The second approach transforms a given input to a logical formula and decides the problem by model counting using a #SAT solver. We discuss the effectiveness and scalability of the two approaches based on the experimental results.

## 4.2. Models

In this section, we introduce a simple relational database model , which will be used in the rest of this chapter. A relational database instance (or simply a database) can be seen as a *table*, of which columns are *attributes*. There are two types of attributes, namely *sensitive* and *nonsensitive* attributes. The values of sensitive attributes are considered as secret, that is, the data owner keeps them confidentially and restrictively and protects them from unauthorized accesses.

**Definition 4.2.1.** A relational database schema (or simply a schema) is a finite set of attributes. Let  $\mathbf{R} = \{A_1, \ldots, A_n\}$  be a schema. We assume that for each attribute  $A_i$   $(1 \leq i \leq n)$ , a finite set of values, denoted by  $dom(A_i)$  is associated. A tuple (or a record) over **R** is  $t = (d_1, \ldots, d_n)$  where  $d_i \in dom(A_i)$  for each  $1 \leq i \leq n$ . Let  $t[A_i] = d_i$ , which is called the value of attribute  $A_i$  in t. That is,  $t = (t[A_1], \ldots, t[A_n])$ . A relational database instance (or simply an instance) of R is a finite set of tuples over R. An instance is sometimes called a table. Let  $I[\mathbf{R}]$  denote the set of all instances of **R**.

Let  $\bf{R}$  be a schema. We assume that  $\bf{R}$  is divided into two disjoint subsets, namely, Se and  $\mathcal{N}S_{e}$ , which are the set of *sensitive attributes* and the set of nonsensitive attributes, respectively. We furthermore assume that a subset  $Qi \subseteq$  $\mathcal{N}$ Se of nonsensitive attributes is given as a *quasi-identifier* of **R**. Note that

the domain of every attribute is finite. We intend that the values of the quasiidentifier can be potentially used to identify the values of the sensitive attributes by linking the attribute values of the quasi-identifier with external data sets.

We define projection, selection and join in the usual way. Let  $R$  be a schema. For a subset of attributes  $\alpha = \{A_{j_1}, \ldots, A_{j_m}\} \subseteq \mathbb{R}$  and a tuple t over  $\mathbb{R}$ , let  $\pi_{\alpha}(t)$  denote the tuple  $(t[A_{j_1}], \ldots, t[A_{j_m}])$ , which is called the projection of t on  $\alpha$ . Also, for an instance  $T \in I[\mathbf{R}]$ , let  $\pi_{\alpha}(T) = {\pi_{\alpha}(t) | t \in T}$ . Let  $T_1 \in I(\mathbf{R}_1)$ and  $T_2 \in I(\mathbf{R}_2)$ . For a filtering condition F, and an instance T, let  $\sigma_F(T)$  denote the set of tuples in T that satisfy F. The natural join of  $T_1$  and  $T_2$  is the instance obtained by "linking" every possible pair of tuples in  $T_1$  and  $T_2$ :

$$
T_1 \bowtie T_2 = \{ t \text{ over } \mathbf{R}_1 \cup \mathbf{R}_2 \mid \text{for some } u \in T_1, \n w \in T_2, t[\mathbf{R}_1] = u \text{ and } t[\mathbf{R}_2] = w \}. \tag{4.1}
$$

Since the natural join operator is associative and commutative, we sometimes view the natural join as a polyadic operator and write  $T_1 \bowtie ... \bowtie T_m$ .

## 4.3. Proposed Framework

In order to provide the definitions, we need to introduce the candidate set of instances of which results of queries are the same as those of the real instance. For a given instance T and queries  $q_1, ..., q_m$ , let  $cand(q_1, ..., q_m, T)$  be the set consisting of all instances that give the same result as T with respect to all queries  $q_1, ..., q_m$ :

$$
cand(q_1, ..., q_m, T) = \{T' \in I(\mathbf{R}) \mid \forall i (1 \le i \le m) \cdot q_i(T) = q_i(T')\}.
$$
\n(4.2)

Each  $T' \in cand(q_1, ..., q_m, T)$  is called a candidate instance.

Let T be a database instance over schema R. T is query-based  $\ell$ -diverse if for each maximal subset of a candidate instance of which tuples have the same quasi-identifier, there are  $\ell$  or more different values of the sensitive attributes.

Suppose that the following information is available to public: a database schema **R**, authorized queries  $q_1, ..., q_m$ , quasi-identifier  $Qi$ , sensitive attributes

Se and a threshold  $\ell$  (a positive integer). Let T be an instance of **R**. An attacker infers sensitive information by taking the natural join of the results of the authorized queries  $q_1, ..., q_m$  on the instance T to obtain the candidate set of sensitive information. We now show three options for the definition of query-based  $\ell$ -diversity as follows.

**Definition 4.3.1.** An instance  $T \in I(\mathbf{R})$  is (query-based)  $\ell$ -diverse (with respect to  $\mathbf{R}, Q_i, Se, q_1, ..., q_m)$ 

(Option 1) if for every  $t \in \pi_{Qi}(T)$ ,

$$
|\{\pi_{Se}(t')\mid \exists T' \in cand(q_1, ..., q_m, T) \cdot
$$

$$
(\pi_{Qi}(t') = t \land t' \in T')\}| \ge l,
$$
\n(4.3)

(Option 2) if for every  $t \in \pi_{Qi}(T)$ , there is an instance  $T' \in cand(q_1, ..., q_m, T)$ such that

$$
|\{\pi_{Se}(t') \mid (\pi_{Qi}(t') = t \land t' \in T')\}| \ge l,
$$
\n(4.4)

(Option 3) if there is  $T' \in cand(q_1, ..., q_m, T)$  such that for every  $t \in \pi_{Qi}(T)$ ,

$$
|\{\pi_{Se}(t') \mid (\pi_{Qi}(t') = t \land t' \in T')\}| \ge l. \tag{4.5}
$$

By definition,  $(4.5)$  implies  $(4.4)$ , and  $(4.4)$  implies  $(4.3)$ .

A conjunctive query consists of projection, selection and join. In our proposed framework, we assume self-join free conjunctive queries.

**Definition 4.3.2.** A query q on **R** is monotonic if for any  $T_1, T_2 \in I(\mathbf{R}), T_1 \subseteq T_2$ implies  $q(T_1) \subseteq q(T_2)$ .

Lemma 4.3.3. Every conjunctive query is monotonic.

If we restrict the class of queries to self-join free conjunctive queries, all the three definitions of  $\ell$ -diversity become equivalent as stated in the next theorem.

**Theorem 4.3.4.** If we assume self-join free conjunctive queries, then the three options in definitions 4.3.1 become equivalent.

Proof. By the following properties 1 and 2.

**Property 1.** For any instances  $T_1, T_2$  and a self-join free conjunctive query q,

$$
q(T_1 \cup T_2) = q(T_1) \cup q(T_2).
$$

*Proof.* Let  $T_1, T_2$  be instances and q be a self-join free conjunctive query. By Lemma 4.3.3, q is monotonic and hence  $q(T_1 \cup T_2) \supseteq q(T_1) \cup q(T_2)$  holds. Since q does not contain self-join,  $q(T_1 \cup T_2) \subseteq q(T_1) \cup q(T_2)$  also holds.  $\Box$ 

**Property** 2. Let T be an instance and  $q_1, \ldots, q_m$  be self-join free conjunctive queries. The largest candidate set in  $cand(q_1, ..., q_m, T)$  (with respect to set inclusion) is the union of all instances in  $cand(q_1, ..., q_m, T)$ .

*Proof.* Let  $T_c = \bigcup_{T' \in cand(q_1,...,q_m,T)} T'$ . By Property 1,

$$
q_i(T_c) = \bigcup_{T' \in cand(q_1, \dots, q_m, T)} q_i(T')
$$
  
= 
$$
\bigcup_{T' \in cand(q_1, \dots, q_m, T)} q_i(T)
$$
  
= 
$$
q_i(T) \ (1 \leq i \leq m).
$$

Hence,  $T_c \in cand(q_1, ..., q_m, T)$ . Apparently,  $T_c$  is the largest set in  $cand(q_1, ..., q_m, T)$ .  $\Box$ 

We define the query-based  $\ell$ -diversity problem as follows:

Input : A schema **R**, an instance  $T \in I(\mathbf{R})$ , authorized queries  $q_1, ..., q_m$ , quasiidentifier  $Qi \subseteq \mathbf{R}$ , sensitive attributes  $Se \subseteq \mathbf{R}$ , and a threshold  $l \geq 1$ .

Output : T is query-based  $\ell$ -diverse or not with respect to **R**,  $Qi$ ,  $Se$ ,  $q_1, ..., q_m$ .

# 4.4. Verification by Relational Algebra

In this section, we describe our verification algorithm that solves the query-based  $\ell$ -diversity problem. For simplicity, we only focus on projection queries. The algorithm can be extended to deal with join without self-join. However, selection cannot be allowed. Also, we assume that the set of sensitive attributes is not empty.

We assume that an attacker knows the domain of each attribute in **, specially** the domains of the sensitive attributes, so that he can infer a candidate instance by adding values of the sensitive attributes chosen from the domain even if (some of) the sensitive attributes are missing in the result of queries  $q_1, ..., q_m$ .

Our algorithm consists of four steps as follows:

1. Obtain the candidate set of tuples  $T'$  by taking the natural join of all results  $q_1(T), ..., q_m(T)$  as follows.

$$
T' = q_1(T) \bowtie \dots \bowtie q_m(T).
$$

2. Let  $Qi' (\subseteq Qi)$  be the set of quasi-identifier that exist in T'. Compute the subset  $T_c$  of  $T'$  consisting of tuples whose quasi-identifier value belongs to the original instance T.

$$
T_c = T' \bowtie \pi_{Qi'}(T).
$$

- 3. Divide  $T_c$  into subsets (equivalence classes)  $g_1, \ldots, g_h$  such that
	- (a)  $\pi_{Qi}(t) = \pi_{Qi}(t')$  for any  $t, t' \in g_i$   $(1 \le i \le h)$  and
	- (b)  $\pi_{Q_i}(t) \neq \pi_{Q_i}(t')$  for any  $t \in g_i$   $(1 \leq i \leq h)$  and  $t' \in g_j$   $(1 \leq j \leq h)$ with  $i \neq j$ .
- 4. Let  $mis\_{Se}$  be the set of sensitive attributes that does not exist in  $T'$ . With  $mis\_Se$  and the threshold  $\ell$ , decide whether T is  $\ell$ -diverse by examining the following necessary and sufficient condition for  $\ell$ -diversity:  $\forall g_i \ (1 \leq i \leq h),$

$$
|g_i| \times \prod_{a \in mis\_{Se}} |dom(a)| \ge \ell. \tag{4.6}
$$

In the last step, the number of different sensitive values in each equivalence class  $g_i$   $(1 \leq i \leq h)$  is computed by using the domains of the missing sensitive attributes mis Se. If for every equivalence class  $g_i$   $(1 \leq i \leq h)$ , the left-hand side of  $(4.6)$  is greater than or equal to the threshold  $\ell$ , the algorithm answers that the given input is  $\ell$ -diverse. If there is at least one equivalence class  $g_i$  such that the left-hand side of (4.6) is less than  $\ell$ , the algorithm answers that the given input is not  $\ell$ -diverse. For a given database instance with n tuples and authorized queries  $q_1, ..., q_m$ , this decision algorithm takes  $O(n^m)$  time.

# 4.5. Verification by Model Counting

In this section, we provide another method for deciding the query-based  $\ell$ -diversity. The method transforms a given input of the problem to a logical formula presented in CNF and decides the problem by model counting using a #SAT solver to count the different values of sensitive attributes in the equivalence classes. This counting procedure terminates when it detects that the number of different values of sensitive attributes of an equivalence class is less than  $\ell$ . The advantage of this method is that it can handle self-join free conjunctive queries, consisting of projection, selection and join without self-join. Henceforth, we assume queries in the class.

Before we explain our method, we give some definitions. For a formula  $\Psi$ , let  $\#models(\Psi)$  denote the number of different models (assignments to variables that make  $\Psi$  true). If a formula  $\Psi$  contains only variables in  $\Sigma$ , we call  $\Psi$  a  $\Sigma$ formula. For a ∑-formula  $\Psi$  and  $\Delta \subseteq \Sigma$ , let  $\Psi_{|\Delta}$  denote the strongest  $\Delta$ -formula implied by  $\Psi$  when considered as a  $\Sigma$ -formula where A is stronger than B if and only if  $A \Rightarrow B$  holds. We say that  $\Psi_{\vert \Delta}$  is the projection of  $\Psi$  onto  $\Delta$ .

Assume that a schema **R** where  $n = |\mathbf{R}|$ , an instance  $T \in I(\mathbf{R})$ , queries  $q_1, \ldots, q_m$  on **R**, quasi-identifies  $Qi \subseteq \mathbf{R}$ , sensitive attributes  $Se \subseteq \mathbf{R}$ , and a threshold  $\ell$  are given. For simplicity, suppose that  $Qi = \{A_1, \ldots, A_k\} \subseteq \mathbf{R}$ , and  $Se = \{A_{k+1}, \ldots, A_m\} \subseteq \mathbf{R}$  where  $1 \leq k < m < n$ . The summary of the method is as follows.

1. Construct a logical formula  $\Phi(x_1, \ldots, x_n)$  such that

 $\Phi(c_1, \ldots, c_n)$  is satisfiable if and only if  $(c_1, \ldots, c_n) \in T_c$  (\*)

Note that  $\phi(x_1, \ldots, x_n)$  has free variables other than  $x_1, \ldots, x_n$  in general.

2. Decide if for all tuple  $(c_1, \ldots, c_k) \in \pi_{Q_i}(T)$ ,

$$
\#models(s(\Phi_p(x_{k+1},\ldots,x_n)|_{X_s})\geq l.
$$

where  $X_s = \{x_{k+1}, \ldots, x_m\}$  and

$$
\Phi_p(x_{k+1},...,x_n) = \Phi(c_1,...,c_k,x_{k+1},...,x_n).
$$

#### Constructing Constraint

Let  $n = |\mathbf{R}|$  and  $n_i = |\mathbf{R_i}|$  where  $\mathbf{R_i}$  is the output schema of  $q_i$   $(1 \leq i \leq m)$ . To construct a formula  $\Phi(x_1, \ldots, x_n)$  satisfying  $(*)$ , we first construct subformulas  $\phi_{q_i}$  and  $O_{q_i}$  for  $1 \leq i \leq m$ .

(1-i) For  $1 \leq i \leq m$ ,  $\phi_{q_i}$  represents the input-output relation of the query  $q_i$ . The formula  $\phi_{q_i}$  contains free variables  $x_1, \ldots, x_n, y_1, \ldots, y_{n_i}$  and satisfies:

for any 
$$
t = (c_1, \ldots, c_n)
$$
 and  $t'_i = (d_1, \ldots, d_{n_i}),$   
 $\phi_{q_i}(c_1, \ldots, c_n, d_1, \ldots, d_{n_i})$  is satisfiable if and only if  $q_i(\lbrace t \rbrace) \subseteq \lbrace t' \rbrace$ .

(Construction) If  $q = T$  then

$$
\phi_q(x_1,...,x_n,y_1,...,y_n) = \bigwedge_{i=1}^n (x_i = y_i).
$$

projection: If  $q = \pi_{\alpha}(q')$  where  $\alpha = \{A_{j_1}, \ldots, A_{j_{n'}}\},\$ 

$$
\phi_q(x_1, \ldots, x_{n_I}, z_1, \ldots, z_{n'})
$$
  
=  $\phi_{q'}(x_1, \ldots, x_{n_I}, y_1, \ldots, y_{n_O}) \wedge \bigwedge_{i=1}^{n'} (y_{j_i} = z_i).$ 

selection: If  $q = \sigma_F(q'),$ 

$$
\phi_q(x_1,\ldots,x_{n_I},z_1,\ldots,z_{n_O})
$$
\n
$$
= \phi_{q'}(x_1,\ldots,x_{n_I},y_1,\ldots,y_{n_O})
$$
\n
$$
\wedge \left(P_F(y_1,\ldots,y_{n_O}) \Rightarrow \bigwedge_{i=1}^{n_O}(y_i=z_i)\right).
$$

where  $P_F(y_1,\ldots,y_{n_O})$  is a formula representing the filtering condition F of  $\sigma_F$ .

cross product: If  $q = q' * q''$ ,

$$
\phi_q(x_1, \dots, x_{n'_I}, x'_1, \dots, x'_{n''_I}, z_1, \dots, z_{n'_O + n''_O})
$$
\n
$$
= \phi_{q'}(x_1, \dots, x_{n'_I}, y_1, \dots, y_{n'_O})
$$
\n
$$
\wedge \phi_{q''}(x'_1, \dots, x'_{n'_I}, y'_1, \dots, y'_{n''_O})
$$
\n
$$
\wedge \bigwedge_{i=1}^{n'_O} (y_i = z_i) \wedge \bigwedge_{i=1}^{n'_O} (y'_i = z_{n'_O + i}).
$$

 $(1-i)$   $O_{q_i}$  is defined as

$$
O_{q_i}(y_1, ..., y_{n_i})
$$
  
=  $\bigvee_{(d_1,...,d_{n_i}) \in q_i(T)} ((y_1 = d_1) \wedge \cdots \wedge (y_{n_i} = d_{n_i})).$ 

 $(1-iii)$  Finally,  $\Phi$  is defined as

$$
\Phi(x_1, ..., x_n) \n= \bigwedge_{i=1}^m (\phi_{q_i}(x_1, ..., x_n, y_{i,1}, ..., y_{i,n_i}) \wedge O_{q_i}(y_{i,1}, ..., y_{i,n_i})).
$$

Remember that in the algorithm of the previous section, we introduce the subsets  $g_1, \ldots, g_h$ , each of which shares same values of the quasi-identifier. For  $g_j$  (1  $\leq j \leq h$ ), let  $(c_1^j)$  $i_1^j, \ldots, c_k^j$  $\binom{d}{k}$  be the values of the quasi-identifier shared by tuples in  $g_j$ . Let  $\Phi_p^j(x_{k+1},\ldots,x_n) = \Phi(c_1^j)$  $i_1^j, \ldots, c_k^j$  $_{k}^{j}, x_{k+1}, \ldots, x_{n}$ ). By  $(*)$ ,  $\Phi_p^j(c_{k+1},\ldots,c_n)$  is satisfiable if and only if  $(c_1^j)$  $i_1^j, \ldots, c_k^j$  $k, c_{k+1}, \ldots, c_n) \in g_j$ . Furthermore,  $\Phi_p^j(x_{k+1},\ldots,x_n)|_{X_s}$  is the strongest Xs-formula implied by  $\Phi_p^j(x_{k+1},\ldots,x_n)$ . Hence, the number of assignments to variables in Xs that make  $\Phi_p^j(x_{k+1},\ldots,x_n)|_{Xs}$ true coincides with the number of different values of Se appearing in tuples that belong to  $g_j$ . Hence, we obtain the following lemma.

**Lemma 4.5.1.** Let **R** be a schema,  $Qi, Se \subseteq \mathbf{R}$  be the quasi-quantifier and sensitive attributes, respectively,  $q_1, \ldots, q_m$  be self-join free conjunctive queries on **R** and  $T \in I(\mathbf{R})$  be an instance. Let  $g_1, \ldots, g_h$  be the subsets of  $T_c$ , each of which shares same values for the quasi-identifier. For each  $j$   $(1 \leq j \leq h)$ , the number of different values of sensitive attributes in  $g_j$  is

$$
\#models(s(\Phi_p^j(x_{k+1},\ldots,x_n)|_{X_s}).
$$

#### Counting Candidates

To count the different values of sensitive attributes for each  $g_j$  ( $1 \leq j \leq h$ ), we transform  $\Phi_p^j(x_{k+1},\ldots,x_n)$  to an equivalent propositional formula  $\Phi_{cnf}^j$  in conjunctive normal form (CNF) by using Sugar [20]. Next, for each  $t' = (c_1, \ldots, c_k) \in$  $\pi_{Qi}(T)$ , we construct a CNF formula  $\psi_{t'}$  that represents  $x_1 = c_1 \wedge \cdots \wedge x_k =$  $c_k$ , and then count  $\#models(\Phi_{cnf}^j \wedge \psi_{t'})_{|P(Xs)}$ , where  $P(Xs)$  is the set of the propositional variables in  $\Phi_{cnf}^j$  corresponding to Xs in  $\Phi_p^j(x_{k+1},\ldots,x_n)$ . We use sharpCDCL [13], which is a  $\#SAT$  solver (an automatic tool for counting the models of a given propositional formula). Among other #SAT solvers that can count models, the advantage of sharpCDCL is that it can automatically count  $\#models(\Psi_{\vert \Delta})$  only by giving a formula  $\Psi$  and a subset  $\Delta$  of propositional variables. If some  $t' \in \pi_{Qi}(T)$  such that  $\#models(\Phi^j_{cnf} \wedge \psi_{t'})_{|P(Xs)} < \ell$  is found, we say that T is not  $\ell$ -diverse. Otherwise, T is  $\ell$ -diverse. For a given propositional formula  $\Phi_{cnf}^j$  over *n* variables, the model counting algorithm counts the distinct truth assignments to variables that make  $\Phi_{cnf}^j$  true, and hence this counting problem is solvable in  $\#P$ .

## 4.6. Experiments

The purpose of the experiment was to investigate the scalability and effectiveness of the algorithms for deciding  $\ell$ -diversity problem.

#### 4.6.1 Experimental Result of Relational Algebra

#### Setup

Experiment were performed on a 3.33 GHz Intel(R) Core(TM) i7 CPU with 6GB of RAM. The operating system was Microsoft Windows 8.1 Enterprise, and implementation was built and run in MySQL Workbench, version 6.1. We used available dataset, Employees Sample Database [23], Copyright (C) 2007, 2008, MySQL AB, version 1.0.6. The database contains about 300,000 tuples with 2.8 million salary entries. In our experiment, the schema consists of ten attributes, where five attributes  ${Gender, DepthName, BirthDate, HireDate, FromDa$ te} were designated as the quasi-identifier and the sensitive attribute is  $\{Salary\}$ .

#### Datasets and Queries

The proposed algorithm was implemented in MySQL and was performed on three instances (datasets) with  $n = 37,500,75,000,150,000,300,000$  tuples. Also, we prepared three queries, each of which is the projection onto the following attributes:

 $q_1$ : { $EmpNo, LastName, Gender$ }.  $q_2$ : { $EmpNo, Salary, HireDate$ }.  $q_3$ : {DeptName}.

In the experiment, we used three sets of queries, namely,  $\mathbf{Q}_A = \{q_1, q_2\}$ ,  $\mathbf{Q}_B =$  ${q_3}$ , and  $\mathbf{Q_C} = {q_1, q_2, q_3}$ . For example, for  $\mathbf{Q_A}$ , the verification algorithm took the natural join of the results of  $q_1$  and  $q_2$  on each of the datasets in Step 1. In Step 3, the algorithm constructed the table from the candidate set  $T_c$  obtained in Step 2 by grouping tuples that have same values of the quasi-identifier. Lastly, in Step 4, the algorithm tested  $\ell$ -diversity ( $\ell = 2$  in the experiment).

Dataset	Cases	Total time
	$q_1, q_2$	7sec
37,500	qз	4sec
	$q_1, q_2, q_3$	11sec
	$q_1, q_2$	19sec
75,000	$q_3$	8sec
	$q_1, q_2, q_3$	31sec
	$q_1, q_2$	1min 7sec
150,000	$q_3$	14sec
	$q_1, q_2, q_3$	1min 47sec
	$q_1, q_2$	4min 8sec
300,000	q3	26sec
	$q_1, q_2, q_3$	8min 48sec

Table 4.1. Total time of verifying 2-diversity.

Table 4.1 shows the total running time of our algorithm. For example, for  $\mathbf{Q}_A$  and the dataset  $n = 37,500$ , the total running time is 7sec. Also the

running time of each set of queries,  $\mathbf{Q}_A, \mathbf{Q}_B$  and  $\mathbf{Q}_C$  on the datasets of size  $n = 37,500,75,000,150,000$  and 300,000. We can observe that the decision algorithm is efficient, in general, and also the computation time depends on the size of the datasets.

#### 4.6.2 Experimental Result of Model Counting

#### Setup

Experiment were performed on a 3.10 GHz Intel(R) Core(TM) i5 CPU with  $8GB$ of RAM. The operating system was Ubantu 14.04. We performed the experiment on a dataset, having 50,000 tuples.

#### Datasets and Queries

In the experiment, we used two instances  $T_1$  and  $T_2$ , having ten and eleven attributes, respectively. Both of  $T_1$  and  $T_2$  have 5,000 tuples. Actually,  $T_1$  was obtained from  $T_2$  by projecting out one of the eleven attributes. We conducted the experiment on the following two settings:

D1: a query  $\sigma_{state=Iwate}(T_1)$ ,  $Qi = \{ID, state\}$  and  $Se = \{Name\}.$ D2: two queries  $\Pi_{BirthYear, BirthMonth}(T_2)$ ,  $\Pi_{BirthYear, BirthMonth}(\sigma_{Carrier=SoftBank}(T_2)),$  $Qi = \{ID, BirthMonth, Carrier\}$ and  $Se = \{BirthYear\}$ .

The experimental results for these settings are shown in Table 4.2 where clauses and variables are those in the transformed CNF formula, projected variables are the variables corresponding to sensitive attributes, min count is the minimum number of different values of sensitive attributes among  $g_1, \ldots, g_h$ . That is, D1 is  $\ell$ -diverse if and only if  $\ell \leq 50$  and D2 is  $\ell$ -diverse if and only if  $\ell \leq 42$ . Next, we increased the number of tuples in  $T_2$  to 10,000, 30,000 and 50,000 and examined the scalability of the proposed method by using the setting D2. The result is shown in Table 4.3. In sharpCDCL, an upperbound  $U$  of the model counting can be specified. That is, when sharpCDCL detects that the current number of models reaches  $U$ , sharpCDCL terminates. The computation times in Table 4.3

			Projected		
	Clauses	Variables	variables	Min count	Time
	34, 271	14,916	4,961	50	6min 15sec
D2	22.941	11 121	96	42	2min 1sec

Table 4.2. Performance of model counting method.

are those when this upperbound is specified as  $U = 20$ . The transformation to

			Projected	
<b>Tuples</b>	Clauses	Variables	variables	Time
5,000	22,941	11, 121	96	1min 58sec
10,000	42, 334	20,877	96	$10\text{min}$ 2sec
30,000	117,557	58,541	106	2hrs 28min 30sec
50,000	187,820	93,633	106	8hrs 56min 8sec

Table 4.3. Scalability of model counting method.

a CNF formula takes less than one second, and model counting dominates the running time.

#### 4.6.3 Additional Result

We compare the results of two approaches by observing the total running time of the smallest dataset and queries in Table 4.5. Additionally, the total running time of the first approach on datasets is shown in Table 4.4.

#### Setup

The experiment were run on a different environment, as mentioned in Section 4.6.2 and 4.6.1, independently.

#### Datasets and Queries

The experiment performed on the same datasets as in Section4.6.1 and queries sets were given as follows, where  ${\bf Q_A} = \{q_4\}$ ,  ${\bf Q_B} = \{q_5, q_6\}$ , and  ${\bf Q_C} = \{q_6, q_7, q_8\}$ .

- $q_4$ : {HireDate, LastName, Gender, Salary}.
- $q_5$ : {HireDate, LastName, Gender}.
- $q_6$ : {HireDate, Salary}.
- $q_7: \{HireDate, LastName\}.$
- $q_8$ : {LastName, Gender}.

Dataset	Cases	Total time
	$q_4$	7sec
37,500	$q_5, q_6$	19sec
	$q_6, q_7, q_8$	$32\sec$
	$q_4$	10 <sub>sec</sub>
75,000	$q_5, q_6$	50 <sub>sec</sub>
	$q_6, q_7, q_8$	$1\text{min}$ 33sec
	$q_4$	1min 16sec
150,000	$q_5, q_6$	$2min\ 54sec$
	$q_6, q_7, q_8$	6min
	$q_4$	4min 28sec
300,000	$q_5, q_6$	$12\text{min }47\text{sec}$
	$q_6, q_7, q_8$	$22\text{min}$ 05sec

Table 4.4. Total running time of relational algebra.

Table 4.5. Computation time of two approaches on a dataset.

Dataset   Cases		Relational algebra   Model counting	
	$q_4$	7sec	3min 32sec
37,500	$q_5, q_6$	19sec	$1\text{min }36\text{sec}$
	$q_6, q_7, q_8$	$32\sec$	38sec

Table 4.5 confirms that verification method for security based on relational algebra using SQL is faster than model counting using #SAT solver.

## 4.7. Conclusion

We have introduced query-based  $\ell$ -diversity as a privacy notion for a realistic database system that assumes access control for queries. This new notion inherits from  $\ell$ -diversity of [15] the quantitative notion for the diversity of sensitive attributes. Also, the notion utilizes  $k$ -secrecy of [11] by taking attacker's inference on the authorized information into consideration.

We proposed two approaches to deciding whether a given database instance satisfies query-based  $\ell$ -diversity with respect to given queries. The first approach is based on relational algebra computation that counts the candidate values of the sensitive attributes. The second approach transforms a given input to a logical formula and then decide the problem by counting models of a formula by a #SAT solver. The first approach can directly be implemented by an existing relational database system such as SQL, and the experimental results show that this approach is fairly efficient. The weakness is that it cannot deal with selection queries. The second approach, on the other hand, can deal with selection queries. However, the model counting in a #SAT solver is generally time consuming and we have not yet customized the solver to our problem and hence, the performance is not good compared with the first approach.

# Chapter 5

# Conclusion

Security against inference attacks has been the main motivation in this dissertation. When releasing data in a database system, it is necessary to prevent the sensitive information of an individual from being disclosed either directly or indirectly. Access control can perfectly control the direct disclosure of the data, but it cannot avoid the indirect disclosure in general. An inference attack is a typical way of indirectly leaking the sensitive information. For defining the security against inference attacks, we need a quantitative notion for security and privacy because the perfect security or privacy cannot be achieved when indirect leakage or inference exists. Quantitative notions such as  $k$ -anonymity and  $\ell$ -diversity were proposed and widely used. However, these notions do not take access control into consideration. The motivation of this dissertation is to propose quantitative notions for security and privacy for a database system with access control.

More concretely, this dissertation has presented two studies on verifying the security against inference attacks on both XML and relational databases.

In Chapter 2, we reviewed quantitative notions for security and privacy, namely, k-anonymity,  $\ell$ -diversity and k-secrecy. The first two are well-known notions mainly for relational databases, which are defined based on the number of tuples or the distribution of the sensitive information included in the tuples indistinguishable from one another. Although these notions are simple and useful, they cannot be directly used in databases having structural data or databases with access control. k-secrecy assumes access control in terms of authorized and unauthorized queries and is defined based on the number of candidates of the results of the unauthorized query (the sensitive information) obtained by an inference attack. k-secrecy is a security notion that can be applied not only to relational databases, but also more general databases such as XML databases. The remaining chapters utilized and extended these notions to investigate verification methods for those extended notions as follows.

Chapter 3 discussed schema-level k-secrecy for XML databases. Based on instance-level k-secrecy introduced by Hashimoto et al., we first defined schemalevel k-secrecy. Schema-level k-secrecy is important because once we can guarantee that a schema is  $k$ -secret, we do not have to check the  $k$ -secrecy of an instance conforming to the schema when an update of a database instance takes place. We showed that  $k$ -secrecy is undecidable for any finite  $k$  when queries are given by linear deterministic top-down tree transducers (LDTTs). Also we showed that  $\infty$ -secrecy is decidable in the same setting. A similar positive result of a stronger query class, linear top-down tree transducers with regular look-ahead  $(LDTT<sup>R</sup>)$ was obtained. We also analyzed the time complexity of the decision problem, and showed that the problem is EXPTIME-complete.

Chapter 4 discussed query-based  $\ell$ -diversity for relational databases. As mentioned above, k-anonymity and  $\ell$ -diversity do not take access control into consideration. This chapter first introduced a new notion, query-based  $\ell$ -diversity by incorporating the way of quantifying the security in defining  $k$ -secrecy into  $\ell$ -diversity. A database instance is (query-based)  $\ell$ -diverse with respect to given authorized queries if there are at least  $\ell$  different values of sensitive information for an individual, which are obtained by inference based on the available query results. It is obvious that query-based  $\ell$ -diversity is decidable for relational databases. In this dissertation, we proposed two approaches to deciding querybased  $\ell$ -diversity, namely, an approach that directly uses relational algebra and an approach based on model counting of a propositional formula. The second approach can deal with self-join free positive conjunctive queries while the first approach cannot deal with queries including selection operations. The scalability and effectiveness of the two approaches were discussed based on the experimental results conducted on the implemented tools. The first approach was implemented by SQL and the second approach was implemented by #SAT solver. For the scalability in efficiency, the experimental results emphasized that the verification by relational algebra was much faster than the other approach. As shown in the additional experimental results, the model counting approach for the dataset with 37, 500 tuples had a total running time nearly identical to the relational algebra approach on the dataset with 75, 000 tuples.

Chapter 3 assumed that the number of authorized queries is one for simplicity. It is left as a future study to extend our algorithm to work with multiple authorized queries and investigate the computational complexity of the extended problem. We also want to extend the notion of schema  $k$ -secrecy to  $k$ -secrecy preservation  $(k$ -SP) through updates. Investigating the decidability of  $k$ -SP in an appropriate model of update operations is interesting future work. Also, applying the approaches proposed in Chapter 4 to other kind of databases such as object-oriented or XML databases is challenging future work. Furthermore, the model counting approach was slower than relational algebra approach. In particular, the model counting approach took very much time when query results were large because the size of the query results directly affects the size of the translated CNF formula. To overcome this problem, we want to improve the translation algorithm so that a given input can be translated into a smaller CNF formula.

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# Publication List

#### Peer-reviewed Journal Paper

1. Chittaphone Phonharath, Kenji Hashimoto, and Hiroyuki Seki, "Deciding Schema k-Secrecy on XML Databases," IEICE Transactions on Information and Systems, vol. E96-D, no. 6, 2013, pp. 1268-1277. (related to Chapter 3)

#### Peer-reviewed International Conference Papers

- 1. Chittaphone Phonharath, Kenji Hashimoto, and Hiroyuki Seki, "Verification of the Security against Inference Attacks on XML Databases," 1st International Workshop on Trends in Tree Automata and Tree Transducers (TTATT), 2012, pp. 11-22. (related to Chapter 3)
- 2. Chittaphone Phonharath, Ryonosuke Takayama, Kenji Hashimoto, and Hiroyuki Seki, "Query-Based  $\ell$ -Diversity," 7th International Conference on Advances in Databases, Knowledge, and Data Applications (DBKDA), 2015, pp. 15-20. (related to Chapter 4)

#### Workshop Papers

- 1. Chittaphone Phonharath, Kenji Hashimoto, and Hiroyuki Seki, "Static Analysis for k-Secrecy against Inference Attacks," IEICE Technical Report SS2011-4, vol. 111, no. 107, 2011, pp. 17-22. (related to Chapter 3)
- 2. Chittaphone Phonharath, Ryonosuke Takayama, Kenji Hashimoto, and

Hiroyuki Seki, "Query-Based  $\ell$ -Diversity," IEICE Technical Report SS2015-14, vol. 115, no. 20, 2015, pp. 65-70. (related to Chapter 4)