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Multi-input Multi-output Feedback Error Learning Control: Theory and Applications

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Abstract

Much attention has been paid recently to Feedback Error Learning (FEL) control, which gives good improvement on the tracking performance of a plant to be controlled by means of on-line learning, without a mathematical model of the plant. A remarkable feature of this scheme is that it uses a feedforward controller which is adjusted by some learning law depending on the feedback error signal. This thesis addresses how to generalize and apply FEL to Multi-input Multi-output (MIMO) systems in the framework of adaptive control.

First, the thesis generalizes the FEL scheme to MIMO systems which are not necessarily biproper, and hence not invertible with properness. An adaptive feedforward controller based on FEL is derived using linear parameterization. The stability of the adaptive control law is proved with and without the positive realness condition for strictly proper and biproper systems, respectively.

Second, a new method for closed-loop identification of a MIMO plant, based on MIMO-FEL, is proposed. Given a roughly designed control system, a feedforward controller is constructed by learning the plant inverse to achieve desirable responses. The trained feedforward controller then generates a model of the plant, which is effective for re-designing the control system to improve performance.

Third, the exponential convergence of the tracking error in a MIMO-FEL scheme is proved without the full persistent excitation condition. The merit of such a result is that, in practice, this condition is undesirable or even impossible to satisfy, while good tracking performance is required. In addition, by making full

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use of this performance, the plant parameter can be estimated during closed-loop operation based on frequency response.

Finally, to prove the practicality of the proposed work an experiment is performed to teach robots how to write one-stroke characters in an actual environment. In particular, a feedforward controller must be designed for two-link manipulators to improve tracking performance despite limited knowledge of the surroundings. MIMO-FEL is employed to tune the feedforward controller. After convergence, the feedforward controllers are switched depending on the target character to be written. This switching criterion is a clear contrast with the precise identification approach, which uses a single general-purpose controller.

Keywords:

Feedback Error Learning, Adaptive and Learning Control, MIMO Systems, Stability Analysis, Closed-loop Identification, Two-link Manipulator

List of publications

Journal Papers

1. B. Alali, K. Hirata and K. Sugimoto, "Generalization of Feedback Error Learning (FEL) to MIMO Systems," Transactions of the Society of Instrument and Control Engineers (SICE), Vol. 43, No. 4, pp. 293-302, 2007.

2. B. Alali, K. Hirata and K. Sugimoto, "Feedback Error Learning for Writing One Stroke Characters by Two-Link Manipulator," Transactions of the Institute of Systems, Control and Information Engineers (ISCIE), Vol. 20, No. 12, pp. 475-481, 2007.

International Conferences (Reviewed)

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2. B. Alali, K. Hirata and K. Sugimoto, "Generalization of Feedback Error Learning (FEL) to Strictly Proper MIMO systems," Proceedings of American Control Conference, Minneapolis, Minnesota, USA, pp. 5451-5456, June, 2006.

3. B. Alali, K. Hirata and K. Sugimoto, "MIMO Feedback Error Learning Control with Stability Proof and Generalizations," 17th International Symposium on Mathematical Theory of Networks and Systems, Kyoto, Japan, pp. 1155-1160, July, 2006.

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5. B. Alali, K. Hirata and K. Sugimoto, “Can a Two-Link Manipulator Learn how to Write?,” IFAC World Congress, Seoul, Korea, July, 2008. (Accepted)

Domestic Conferences

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3. B. Alali, K. Hirata and K. Sugimoto, “Feedback Error Learning with Insufficient Excitation,” SICE 8th Annual Conference on Control Systems CCS2008, Kyoto, Japan, March 5-7, 2008.

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Last but not least, I would like to say sorry for my lovely daughter, Sama, whom I left in my country after she was born to finish this work. Because of her love and miss, I did my best to achieve substantial results to be able to graduate in a short period.

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Chapter 1

Introduction

1.1. Overview

Recently, adaptive learning control has been attracting much attention due to its great effectiveness in improving tracking performance. The powerful idea behind adaptive learning control is that it uses adaptive and learning control techniques to automatically adjust the parameters or structure of the controller to provide a satisfactory or desired control response. It can be classified into two categories: memory-based and memoryless techniques. For the memory-based techniques, which require memory, the most common approach is the so-called iterative learning control (ILC), which uses the error recorded in the preceding trial to improve the performance in the current one and requires, in general, a resetting procedure at the beginning of each new trial [1, 2, 3]. Thus, ILC requires many steps to achieve asymptotic convergence of the tracking error. However, for the memoryless techniques, which do not require memory (*i.e.*, continuous time framework), it is common to use classical adaptive control techniques, which can be divided into direct and indirect approaches, to ensure asymptotic convergence of the tracking error [4, 5, 6, 7]. These techniques usually include the notion of "model" of the true plant or the existence of an identification mechanism to provide an implicit or explicit plant model to the adaptation algorithm. Further, it is known that the adaptation of feedback controller produces a bad transient response [4, 5] due to initial estimation error.

However, there is a different approach to these techniques, which requires neither memory nor many trials nor extensive modeling nor online parameter estimation. This technique is called Feedback Error Learning (FEL). The concept of FEL was given by Kawato and his group in 1987 [8], who proposed a model of our body acquiring accurate motion. Signal transmission in our neural system is too slow to achieve enough accuracy via feedback alone. Hence our body uses a feedforward mechanism and adapts it by learning from feedback error. Kawato *et al.* applied this mechanism to control system design, which was novel in control literature. They originally adopted artificial neural networks (ANN) as a function generator for the feedforward controller. Their pioneering results can be summarized as follows:

1. There is no need for extensive modeling or parameter estimation for the plant to be controlled.
2. As learning proceeds, an inverse dynamics model, which represents the feedforward controller, gradually takes the place of the feedback controller as the main controller.
3. After learning, the acquired model learns the dynamics and inverse dynamics of the plant.
4. The model can adapt to a sudden change in the dynamics of the controlled system.

FEL has become a promising method for designing a control system having a good tracking property without extensive system modeling.

Miyamura and Kimura [9] established the theoretical stability of the FEL in the framework of linear control theory. They investigated FEL from the viewpoint of adaptive control theory. The learning algorithm of FEL has been exploited extensively in the framework of conventional adaptive control. Further, the stability of the algorithm has been established based on the notion of strictly positive realness (SPR), see Appendix for a definition.

However, Muramatsu and Watanabe [10] relaxed the SPR by using the error signal between the reference and the output signal as well as the feedback input.

Unlike [8], references [9, 10] used an adaptive linear filter for the function approximator, and did not use ANN; nonetheless the learning of the inverse model was effective.

Wongsura and Kongprawechnon [11, 12, 13, 14] studied a discrete-time setting of FEL and call it discrete-time feedback error learning (DTFEL).

An extension of the above work to systems with known time delay was discussed by Miyamura and Kimura [15], Miyamura [16] and Muramatsu and Watanabe [17]. However, Terashita and Kimura [18] considered FEL with unknown time delay.

FEL was also extended and studied for nonlinear systems [19, 20, 21]. In particular, the relationship between FEL and nonlinear adaptive control with adaptive feedback linearization was discussed by Nakanishi and Schaal [19]. They showed that FEL can be interpreted as a form of nonlinear adaptive control.

FEL has been implemented successfully in robotics applications. Miyamoto *et al.* [22] have successfully applied the work proposed in [8] to control an industrial robotic manipulator (Kawasaki-Unimate PUMA 260) with the neural networks model in a microcomputer. It has been also applied to SCARA robots [23, 24], and been implemented in humanoid robot, inverted pendulum and flexible link [25, 26, 27]. All the experimental results have shown the effectiveness of using FEL to improve tracking performance.

The following definitions are used throughout this dissertation:

- A system $G(s)$ is strictly proper if $G(s) \to 0$ as $s \to \infty$.
- A system $G(s)$ is bi-proper if $G(s) \to C$, where $\det C \neq 0$, as $s \to \infty$.
- A system $P(s)$ is improper if $G(s) \to \infty$ as $s \to \infty$.
- A signal is sufficiently rich if it satisfies the persistent excitation (PE) condition, see Appendix.

1.2. The FEL Scheme

The original structure of FEL is shown in Fig. 1.1. It is a two degree of freedom (2DOF) control scheme which consists of a feedback controller and a tunable feed-

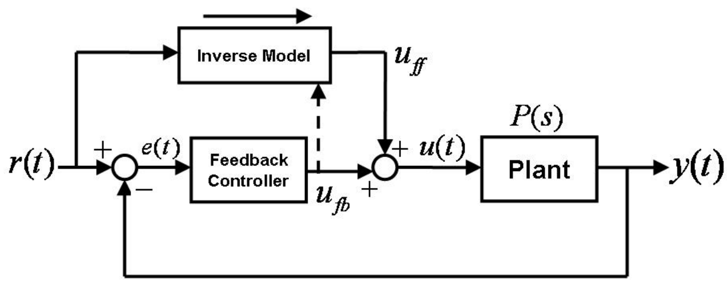

Figure 1.1. Feedback Error Learning (Original Scheme, Kawato [28])

forward controller corresponding to the inverse model. The feedback controller maintains closed-loop stability while the feedforward controller improves tracking performance. ANN models have been used to represent the inverse model structure [8, 28]. Recently, linear filter parameterizations have been used instead of ANN to parameterize the inverse model structure [9, 10]. The main objective of FEL scheme is to use feedback control input $u_{fb}(t)$ to tune the feedforward controller (inverse model). After convergence, the feedforward controller equals the inverse model of the plant and $e(t)$ converges to zero.

The learning law for the above FEL scheme established in [8] can be described as follows:

$$\frac{d\theta}{dt} = \alpha u_{\mathrm{fb}}(t)\frac{\partial u_{\mathrm{ff}}}{\partial \theta}(t), \tag{1.1}$$

where θ is the tunable parameter of the feedforward controller (inverse model), and α is to adjust the learning speed. Miyamura and Kimura [9] derived the learning law based on adaptive control standpoint using the gradient method with the error function defined as

$$V = \frac{1}{2}(u_{\mathrm{ff}}(t) - u_0(t))^T(u_{\mathrm{ff}}(t) - u_0(t)), \tag{1.2}$$

where u_0 is the exact feedforward signal which gives $y(t) = r(t)$. The gradient method deduces the following tuning rule,

$$\frac{d\theta}{dt} = -\alpha\frac{\partial u_{\mathrm{ff}}}{\partial \theta}(t)(u_{\mathrm{ff}}(t) - u_0(t)). \tag{1.3}$$

However, this tuning rule is not feasible because u_0 is unknown. Under the assumption that the plant input $u(t)$ is correct, *i.e.*,

$$u(t) = u_0(t) = P^{-1}(s)r(t), \tag{1.4}$$

then the tuning rule becomes

$$\frac{d\theta}{dt} = \alpha \frac{\partial u_{\text{ff}}}{\partial \theta}(t) u_{\text{fb}}(t). \tag{1.5}$$

The stability of this deriving rule has been studied in detail with the SPR condition in [9] and without SPR in [10].

1.3. Research Goals

Adaptive learning control of Multi-input Multi-output (MIMO) systems is an important research area with both theoretical challenges and practical significance (*e.g.*, process control systems, robotics control systems and aircraft control systems usually have multiple inputs and multiple outputs). The main technical difficulty in multivariable adaptive learning control is to deal with dynamic interactions between system inputs and outputs. The present work overcomes this difficulty through studies on how to generalize and apply FEL to MIMO system from the perspective of adaptive control.

The objectives of the present thesis are four-fold. First, learning control structures are studied for a MIMO system using FEL. By using linear system parameterization as a function approximator of the feedforward control, I derive a learning law to adjust the inverse of the plant. A theoretical treatment of how to generalize FEL to MIMO systems is presented in the framework of adaptive control.

Second, I propose a new method for closed-loop identification of a MIMO plant. Given a roughly designed control system, a feedforward controller is constructed by learning the plant inverse to achieve desirable responses. The trained feedforward controller then generates a model of the plant, which is effective for re-designing the control system to improve performance.

Third, I study the exponential convergence of the tracking error in a MIMO-FEL scheme having insufficient excitation. The merit of such a result is that good tracking performance is only required in practical applications while it is not desirable or even impossible to satisfy the PE condition. In addition, by making full use of this performance, I estimate the plant parameter during closed-loop operation based on frequency response.

Finally, I consider a version of the problem of how to teach robots to write characters in an actual environment. In particular, I design a feedforward controller for two-link manipulators to improve tracking performance despite limited knowledge of the surroundings. MIMO-FEL is employed to achieve the objective.

1.4. Contributions

The main contribution of this research can be summarized as follows

1. A theoretical treatment of how to generalize FEL to MIMO systems has been studied in the framework of adaptive control.

2. A new method for closed-loop identification of a MIMO plant based on the MIMO-FEL scheme has been proposed.

3. Exponential convergence of the tracking error in the MIMO-FEL scheme having insufficient excitation has been proved. Further, parameter estimation of the plant based on FEL has been proposed.

4. Experimental validity of the MIMO-FEL scheme is performed using a two-link manipulator.

1.5. Outline

This dissertation is divided into five main chapters. Chapter 2 first generalizes the FEL scheme to MIMO systems which are not necessarily biproper, and hence not invertible with properness. An adaptive feedforward controller based on FEL is derived using linear parameterization. The stability of the adaptive control law is proved with and without the SPR condition for strictly proper and biproper systems, respectively.

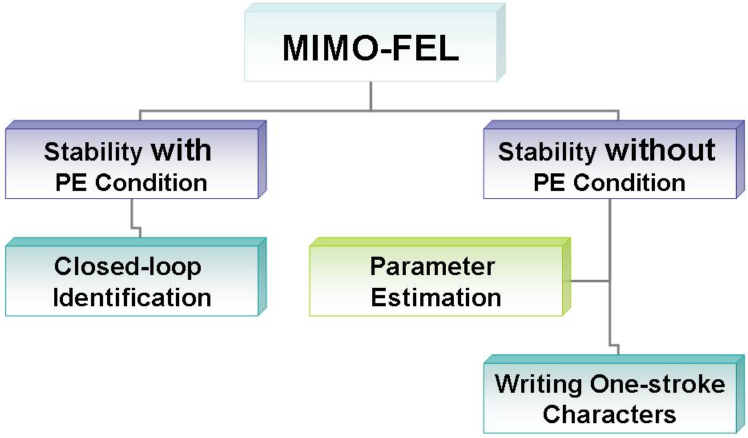

Figure 1.2. Dissertation Outline

Chapter 3 then considers a new method for closed-loop identification of a MIMO plant based on MIMO-FEL based on the PE condition. Given a control system roughly designed, a feedforward controller is constructed by learning to achieve desirable responses. Using sufficiently rich reference signal, the trained feedforward controller gives a model of the plant.

Chapter 4 studies the exponential convergence of the tracking error in MIMO-FEL scheme having insufficient excitation; without satisfying the PE condition. In addition, by making full use of this performance, the plant parameter can be estimated while in closed-loop operation based on frequency response. This chapter basically complements the results obtained in Chapters 2 and 3.

Chapter 5 considers how to implement the results obtained in the previous chapters to a real-world problem to prove the practicality of the proposed work. A problem of how to teach robots to write characters in an actual environment is considered. In particular, a feedforward controller must be designed for two-link manipulators to improve tracking performance despite limited knowledge of the surroundings. MIMO-FEL is employed to achieve the control objective.

Finally, Chapter 6 summarizes the main achievements of this doctoral dissertation and proposes possible extensions and directions for future work. The outline is summarized in Fig. 1.2.

Chapter 2

Generalization of FEL to MIMO Systems

This chapter studies MIMO learning control systems. It generalizes the FEL scheme to MIMO systems which are not necessarily biproper, and hence not invertible with properness. Assuming that a diagonal interactor is known, a pre-filter and an exact feedforward controller are designed to achieve FEL. Furthermore, an adaptive feedforward controller based on FEL is derived using linear parameterization. The stability of the adaptive control law is proved with and without positive realness condition for strictly proper and biproper systems, respectively. Simulation results illustrate the effectiveness of the proposed method.

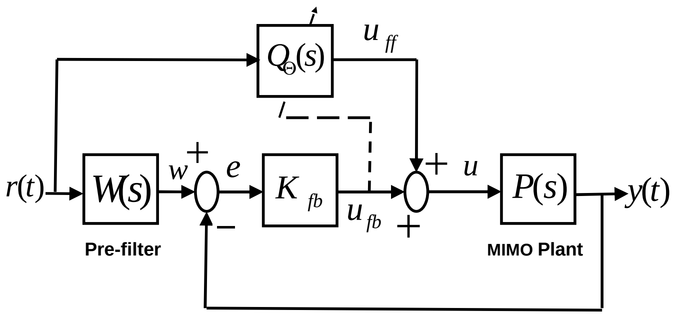

Figure 2.1. MIMO-FEL Architecture

2.1. Introduction

The main objective of FEL is to implement the feedforward controller as a function approximator, improving approximation on-line, instead of designing linear control on the basis of a plant model. The types of function approximators used in [8] and [29] as examples are Multi Layer Perception (MLP) and Cerebellar Model Articulation Controller (CMAC), respectively.

Several problems exist for this kind of function approximator, such as slow convergence, local minima, computational load and high memory requirement [30]. However, applications have shown that the FEL controller gives a considerable improvement on the performance of the system [8, 23, 28, 29, 31]. The results in the literature have shown that FEL can improve the tracking of the desired trajectory significantly without extensive modeling. FEL uses feedback error as a learning signal, which is essentially new in control literature [9, 10].

Recently, Miyamura and Kimura [9] have established a control theoretical validity of the FEL method in the frame of adaptive control, proving its stability based on strictly positive realness, whereas Muramatsu and Watanabe [10] have relaxed the positive realness condition of FEL by using the error signal between the reference and the output signal as well as the feedback input. Unlike [8], references [9, 10] have used an adaptive linear filter for a function approximator, and not MLP or CMAC, but the learning of the inverse model was nevertheless effective.

The objectives of this chapter are two-fold. First, because most control applications are MIMO, I generalize FEL for a MIMO system. The stability of the derived MIMO-FEL learning law for square case is proved by using the idea of linear parameterization, which was given by [9, 10] for single-input single-output (SISO) systems. The parameter has a matrix form instead of a vector form, but I could derive a learning law in a similar way as in the SISO case.

Second, I also relax the biproper condition even in the MIMO case. The concept of the interactor (Wolovich and Falb [32]) is used for this purpose. This concept was originally introduced to generalize the relative degree of scalar transfer functions to MIMO systems.

2.2. Problem Formulation

Consider an n-dimensional controllable and observable linear system

$$\dot{x}(t) = Ax(t) + Bu(t),\ \ y(t) = Cx(t) + Du(t) \tag{2.1}$$

with m-inputs and m-outputs (*i.e.*, square system). Assume that the system is minimum phase. Namely, I assume

$$\det \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix} \neq 0,\ \text{ for all } \mathrm{Re}(s) \geq 0. \tag{2.2}$$

If, further, $\det D \neq 0$, then the system is invertible with properness and hence generalization of FEL is straightforward, so that the method below is significantly simplified. On the other hand, if $\det D = 0$, then the generalization is difficult, since I need to define an appropriate pre-filter. I will overcome the difficulty below. To explain the method clearly, I will further assume that $D = 0$ in what follows. (If $\det D = 0$ and $D \neq 0$, then the argument will be more complicated but the basic idea still holds.)

Therefore, the transfer matrix is $P(s) = C(sI - A)^{-1}B$. I also assume that A, B, C are unknown, but that

$$\Lambda := \begin{bmatrix} c_1 A^{\mu_1 - 1} B \\ \vdots \\ c_m A^{\mu_m - 1} B \end{bmatrix} \tag{2.3}$$

is invertible, where c_k is the k^{th} row vector of C and μ_k is the minimal integer such that

$$c_k A^{\mu_k - 1} B \neq 0, \mu_k \geq 1. \tag{2.4}$$

The integers $\mu_1, \mu_2, \cdots, \mu_m$ can be regarded as a generalization of the relative degree. I assume that these integers are known. Furthermore, they have a close relationship to the following concept (Wolovich and Falb [32]).

Definition 2.1: Given $P(s)$, a square polynomial matrix $L(s)$ is called an interactor if

$$\lim_{s \to \infty} L(s)P(s) \tag{2.5}$$

is a finite matrix with full rank.

With few loss of generality, I can set the interactor matrix to a diagonal form

$$L(s) = \begin{bmatrix} (s+a_1)^{\mu_1} & & 0 \\ & \ddots & \\ 0 & & (s+a_m)^{\mu_m} \end{bmatrix} \tag{2.6}$$

where μ_k is as defined before, and a_k is an arbitrarily chosen positive real number for $k = 1, \cdots, m$. Then (2.5) equals Λ in (2.3).

The feedback error learning architecture of the MIMO system is shown in Fig. 2.1. The objective of the control is to minimize the error signal between $w(t)$ and the plant output $y(t)$. The following equations can be deduced from Fig. 2.1:

$$\begin{array}{rcl} y(t) & = & P(s)u(t) \\ u(t) & = & u_{fb}(t) + u_{ff}(t) \\ u_{fb}(t) & = & K_{fb}e(t),\ e(t) = w(t) - y(t) \\ w(t) & = & W(s)r(t) \\ u_{ff}(t) & = & Q_{\Theta}(s)r(t), \end{array} \tag{2.7}$$

where K_{fb} is a feedback gain to stabilize the plant, and Θ is a tunable parameter. The first equation implies

$y(t) = \pounds^{-1}[P(s)](t) * u(t),$

where $*$ denotes the time-domain convolution. For the sake of simplicity, I will adopt this kind of abuse in time and frequency domains throughout this thesis. I first note that, if the system is invertible with properness as in [9, 10], then I can readily take $Q_\Theta = P^{-1}$ and obtain a perfect tracking. This is not the case, however, since I have assumed that $P(s)$ is strictly proper, and hence $P^{-1}(s)$ becomes improper. In the next section, I will see how this difficulty is overcome by means of a pre-filter $W(s)$ and the concept of the interactor.

2.3. Proposed Method

I now introduce exact and adaptive methods for finding the inverse of the system. First, I set up $Q(s)$ for the exact case using the interactor concept [32]. Second, I establish an adaptive method to adjust the parameter Θ in $Q_\Theta(s)$ of equation (2.7) when A, B, C of the plant are assumed to be unknown. After that, I combine both methods to derive a tuning rule using the gradient descent concept.

2.3.1 Exact Feedforward Controller

For the original system (2.1) and the interactor $L(s)$ in (2.6) with arbitrarily chosen $a_k (k = 1, \cdots, m)$, there exists a gain R such that

$$N(s) = C(sI - A + BR)^{-1}B = L(s)^{-1}\Lambda. \tag{2.8}$$

The gain R can be obtained by the simple formula (Mutoh [33]):

$$R := \Lambda^{-1}[L_0, \cdots, L_\mu]\begin{bmatrix} C \\ CA \\ \vdots \\ CA^\mu \end{bmatrix} \tag{2.9}$$

where

$$L(s) = L_0 + L_1 s + \cdots + L_\mu s^\mu, \tag{2.10}$$

$$\mu := \max(\mu_1, \cdots, \mu_m). \tag{2.11}$$

Take

$$
\begin{aligned}
W(s) &= L(s)^{-1} \\
&= \begin{bmatrix} \frac{1}{(s+a_1)^{\mu_1}} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{(s+a_m)^{\mu_m}} \end{bmatrix}
\end{aligned} \tag{2.12}
$$

Based on the above, I derive the following lemma.

Lemma 2.1: If the feedforward controller $Q(s)$ is defined by

$$Q(s) = [I - R(sI - A + BR)^{-1}B]\Lambda^{-1}. \tag{2.13}$$

then

$$P(s)Q(s) = W(s). \tag{2.14}$$

Proof:

$$
\begin{aligned}
&P(s)Q(s) - W(s) \\
&= C(sI - A)^{-1}B[I - R(sI - A + BR)^{-1}B]\Lambda^{-1} - C(sI - A + BR)^{-1}B\Lambda^{-1} \\
&= C\{(sI - A)^{-1}B - (sI - A)^{-1}BR(sI - A + BR)^{-1}B - (sI - A + BR)^{-1}B\}\Lambda^{-1} \\
&= C\{(sI - A)^{-1}B - [(sI - A)^{-1}BR + I] \cdot [sI - A + BR]^{-1}B\}\Lambda^{-1} \\
&= C\{(sI - A)^{-1}B - (sI - A)^{-1}[BR + sI - A] \cdot [sI - A + BR]^{-1}B\}\Lambda^{-1} \\
&= C\{(sI - A)^{-1}B - (sI - A)^{-1}B\}\Lambda^{-1} = 0. \ \diamond
\end{aligned}
$$

Lemma 1 means that the cascade connection (2.14) makes the response as simple as possible. $W(s)$ in (2.12) represents invertible delay due to strict properness of the plant. I thus take as the exact feedforward control:

$$u_0(t) = Q(s)r(t). \tag{2.15}$$

Since A, B, C of the system are assumed to be unknown, I need to adjust $Q(s)$ as presented in the next section.

Remark: I will use W(s) and Q(s) in (2.12), (2.13) as a pre-filter and a feedforward controller in Fig. 2.1, which is a natural generalization of Section 4 of [9].

2.3.2 Adaptive Feedforward Controller

In this section, I generalize FEL for a MIMO system by using adaptive observer parameterization [4]. First, I consider the following dynamical system to parameterize the feedforward controller.

$$\begin{aligned} \dot{\eta}_1(t) &= A_f\eta_1(t) + B_f r(t) \\ \dot{\eta}_2(t) &= A_f\eta_2(t) + B_f u_0(t) \\ u_0(t) &= F_0\eta_1(t) + G_0\eta_2(t) + H_0 r(t) \\ &= \Theta_0\eta(t), \end{aligned} \tag{2.16}$$

where

$$\Theta_0 = [F_0 \;\; G_0 \;\; H_0], \eta = \begin{bmatrix} \eta_1(t) \\ \eta_2(t) \\ r(t) \end{bmatrix}, \tag{2.17}$$

det $H_0 \neq 0$. F_0, G_0, H_0 are unknown matrices to be estimated.

I will show that the parameterization of (2.16) can yield an arbitrary transfer matrix from $r(t)$ to $u_0(t)$. Take A_f and B_f in a controllable canonical form as shown in (2.18) and (2.19),

$$A_f = \begin{bmatrix} \begin{matrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ & & \cdots & 1 \\ -f_\nu & & & -f_1 \end{matrix} & & 0 \\ & \ddots & \\ 0 & & \begin{matrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ & & \cdots & 1 \\ -f_\nu & & & -f_1 \end{matrix} \end{bmatrix} \tag{2.18}$$

$$B_f = \begin{bmatrix} 0 & & \\ \vdots & & 0 \\ 1 & & \\ & \ddots & \\ & & 0 \\ 0 & & \vdots \\ & & 1 \end{bmatrix}, \tag{2.19}$$

where A_f is a stable matrix and (A_f, B_f) is controllable. Then I have

$$\begin{aligned} u_0 &= (I - G_0(sI - A_f)^{-1}B_f)^{-1} \cdot (H_0 + F_0(sI - A_f)^{-1}B_f)r(t) \\ &=: Q_{\Theta_0}(s)r(t). \end{aligned} \tag{2.20}$$

Taking into account that

$$(sI - A_f)^{-1}B_f = S(s)\frac{1}{s^\mu + f_1 s^{\mu-1} + \cdots + f_\mu} \tag{2.21}$$

where

$$S(s) = \begin{bmatrix} 1 & & \\ s & & 0 \\ \vdots & & \\ s^{\mu-1} & & \\ & \ddots & \\ & & 1 \\ 0 & & s \\ & & \vdots \\ & & s^{\mu-1} \end{bmatrix}, \tag{2.22}$$

I thus obtain

$$u_0(t) = [X(s)]^{-1}Y(s)r(t), \tag{2.23}$$

where

$$X(s) = [(s^\mu + f_1 s^{\mu-1} + \cdots + f_\mu)I - G_0 S(s)],$$

$$Y(s) = [(s^\mu + f_1 s^{\mu-1} + \cdots + f_\mu)H_0 + F_0 S(s)].$$

It can be shown that equation (2.23) gives a complete representation of any transfer matrix of this size and dimension. Namely, by taking F_0, G_0, H_0 appropriately, I can obtain $Q(s)$ derived in the previous section, in the form of (2.23). In reality, however, I do not know such parameters F_0, G_0, and H_0, so that I regard these matrices as tunable coefficients and adjust them according to some learning law.

I now construct a tunable feedforward controller. To generate $u_{ff}(t)$, I consider the following dynamical system, instead of (2.16):

$$\begin{aligned}
\dot{\xi}_1(t) &= A_f \xi_1(t) + B_f r(t) \\
\dot{\xi}_2(t) &= A_f \xi_2(t) + B_f u_{ff}(t) \\
u_{ff}(t) &= F(t)\xi_1(t) + G(t)\xi_2(t) + H(t)r(t) \\
&= \Theta(t)\xi(t),
\end{aligned} \tag{2.24}$$

where

$$\Theta(t) = [F(t) \;\; G(t) \;\; H(t)], \;\; \xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ r(t) \end{bmatrix}. \tag{2.25}$$

Note that the unknown parametric matrices $F(t)$, $G(t)$, $H(t)$ enter linearly. The matrix $\Theta(t)$ is tuned using the learning law, which will be derived in the next section.

2.3.3 Learning Law

The learning law is derived using the gradient method with an error function, defined for each time as follows:

$$V = \frac{1}{2}(u_{ff} - u_0)^T(u_{ff} - u_0). \tag{2.26}$$

Then the steepest gradient method is formulated as

$$\frac{d\Theta}{dt} = -\alpha\frac{\partial V}{\partial \Theta}, \tag{2.27}$$

for small $\alpha > 0$.

A major difference from references [8] and [9] is, however, that Θ is not a vector, but a matrix. Nonetheless the idea carries over only with a slight modification.

I define

$$\frac{\partial V}{\partial \Theta} = \left(\frac{\partial V}{\partial \theta_{ij}}\right), \tag{2.28}$$

where

$$\Theta = (\theta_{ij})\,. \tag{2.29}$$

Thus, I calculate

$$\frac{\partial V}{\partial \theta_{ij}} = \frac{\partial}{\partial \theta_{ij}}\left[\frac{1}{2}\sum_k v_k^2\right], \tag{2.30}$$

where

$$v = u_{ff} - u_0 = (v_1, \cdots, v_m)^T. \tag{2.31}$$

Using (2.24), I have

$$v = \Theta\xi - u_0. \tag{2.32}$$

Therefore

$$\begin{aligned} \tfrac{\partial V}{\partial \theta_{ij}} &= \textstyle\sum_k [v_k \tfrac{\partial v_k}{\partial \theta_{ij}}] \\ &= \textstyle\sum_k [v_k \tfrac{\partial}{\partial \theta_{ij}}[\sum_l (\theta_{kl}\xi_l - u_{0k})]]. \end{aligned} \tag{2.33}$$

I also have

$$\frac{\partial}{\partial \theta_{ij}}\sum_l(\theta_{kl}\xi_l - u_{0k}) = \delta_{ik}\xi_j. \tag{2.34}$$

where

$$\delta_{ik} = \begin{cases} 0 & i \neq k \\ 1 & i = k \end{cases}. \tag{2.35}$$

Hence, I have

$$\frac{\partial V}{\partial \theta_{ij}} = v_i \xi_j, \qquad i, j = 1, \cdots, m. \tag{2.36}$$

Therefore, I obtain

$$\frac{\partial V}{\partial \Theta} = v \xi^T. \tag{2.37}$$

I have derived (2.15) since $e(t) = 0$, which implies that I can take $u(t) = u_0(t)$[1]. The tuning rule then becomes

$$\frac{d\Theta}{dt} = -\alpha(u_{ff}(t) - u(t))\xi^T(t) = \alpha u_{fb}(t)\xi^T(t). \tag{2.38}$$

where α is a scalar parameter introduced to adjust the adaptation speed. The above learning law clearly depends on the feedback error signal, and this is why I call it feedback error learning.

2.4. Stability of the Proposed Scheme

The stability of the above differential equation (2.38) can be verified as in [9] using the strictly positive realness condition. Using (2.14), (2.15) and (2.7), I obtain the following relation

$$\begin{aligned} P(s)[u_0(t) - u(t)] &= P(s)Q(s)r(t) - P(s)u(t) \\ &= W(s)r(t) - y(t) = e(t). \end{aligned} \tag{2.39}$$

If I assume that the initial conditions of equations (2.16) and (2.24) are the same, then subtraction between the first two equations of (2.16) and (2.24) yields the following relations

---

[1]This approximation holds in the neighborhood of the exact parameter.

$$\begin{aligned} \eta_1 &= \xi_1, \\ \eta_2 &= \xi_2 + (sI - A_f)^{-1} B_f (u_0(t) - u(t)). \end{aligned} \tag{2.40}$$

Using (2.16), (2.24) and (2.7), I obtain the following relation

$$u_0(t) - u(t) = \Theta_0 \eta(t) - \Theta(t)\xi(t) - K_{fb} e(t). \tag{2.41}$$

Substituting (2.40) in (2.41), I have

$$u_0(t) - u(t) = \Theta_0 \xi(t) - \Theta(t)\xi(t) - K_{fb} e(t) + G_0 (sI - A_f)^{-1} B_f (u_0(t) - u(t)),$$

$$[I - G_0 (sI - A_f)^{-1} B_f](u_0(t) - u(t)) = -[(\Theta(t) - \Theta_0)\xi(t) + K_{fb} e(t)],$$

$$u_0(t) - u(t) = -[I - G_0 (sI - A_f)^{-1} B_f]^{-1} [\Psi(t)\xi(t) + K_{fb} e(t)], \tag{2.42}$$

where

$$\Psi(t) = \Theta(t) - \Theta_0. \tag{2.43}$$

Θ_0 can be regarded as the nominal value of $\Theta(t)$. Pre-multiplying (2.42) by $P(s)$, I have

$$P(s)[u_0(t) - u(t)] = -P(s)[J(s)]^{-1} [\Psi(t)\xi(t) + K_{fb} e(t)], \tag{2.44}$$

where

$$J(s) = [I - G_0 (sI - A_f)^{-1} B_f]$$

From (2.15) and (2.20), I have

$$[I - G_0 (sI - A_f)^{-1} B_f]^{-1} = Q(s) \cdot [H_0 + F_0 (sI - A_f)^{-1} B_f]^{-1}. \tag{2.45}$$

Substituting (2.39) and (2.45) in (2.44), I have

$$e(t) = -P(s)Q(s)[H_0 + F_0(sI - A_f)^{-1}B_f]^{-1} \cdot [\Psi(t)\xi(t) + K_{fb}e(t)]. \tag{2.46}$$

Using (2.14), I obtain

$$\begin{aligned} e(t) &= -[I + W(s)(H_0 + F_0(sI - A_f)^{-1}B_f)^{-1}K_{fb}]^{-1} \\ &\quad \cdot W(s)[H_0 + F_0(sI - A_f)^{-1}B_f]^{-1}\Psi(t)\xi(t) \\ &= -[(H_0 + F_0(sI - A_f)^{-1}B_f)L(s) + K_{fb}]^{-1} \cdot \Psi(t)\xi(t). \end{aligned} \tag{2.47}$$

Substituting $e(t)$ in the derived learning law (2.38), I obtain

$$\tfrac{d\Theta}{dt} = -\alpha K_{fb}[(H_0 + F_0(sI - A_f)^{-1}B_f)L(s) + K_{fb}]^{-1}\Psi(t)\xi(t)\xi(t)^T. \tag{2.48}$$

Namely, I have

$$\frac{d\Psi}{dt} = -\alpha M(s)\Psi(t)\xi(t)\xi^T(t), \tag{2.49}$$

where

$$\begin{aligned} M(s) &= K_{fb}[(H_0 + F_0(sI - A_f)^{-1}B_f)L(s) + K_{fb}]^{-1}, \\ &= K_{fb}W(s)[H_0 + F_0(sI - A_f)^{-1}B_f + K_{fb}W(s)]^{-1} \end{aligned} \tag{2.50}$$

Following the idea in [9], I show the following lemma.

Lemma 2.2: Let $M(s)$ be a strictly positive real transfer matrix and $\xi(t)$ be an arbitrary time-varying vector. Then, the solution $\Psi(t)$ of (2.49) tends to a constant matrix Ψ_0 such that $\Psi_0\xi(t) \to 0$. If $\xi(t)$ satisfies the PE condition, the above Ψ_0 is equal to zero matrix. *See Appendix for PE condition.*

Proof: Consider the state space representation of (2.49)

$$\begin{aligned} \tfrac{dx(t)}{dt} &= \hat{A}x(t) + \hat{B}\Psi(t)\xi(t) \\ y(t) &= \hat{C}x(t) + \hat{D}\Psi(t)\xi(t) \\ \tfrac{d\Psi(t)}{dt} &= -y(t)\xi^T(t). \end{aligned} \tag{2.51}$$

Since $M(s)$ is strictly positive real, it implies that there exist $P = P^T > 0$, T, and Z, and a positive constant ϵ given in the KYP lemma (see Appendix). I consider the following Lyapunov function

$$V(t) = tr\{\Psi^T(t)\Psi(t)\} + x^T(t)Px(t). \tag{2.52}$$

Taking the derivative

$$\begin{aligned} \dot{V}(t) &= tr\{\Psi^T(t)\dot{\Psi}(t)\} + tr\{\dot{\Psi}^T(t)\Psi(t)\} \\ &\quad + \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t). \end{aligned} \tag{2.53}$$

Using the KYP lemma, I have

$$\dot{V}(t) = -\left\|Tx(t) + Z\Psi(t)\xi(t)\right\|^2 - \epsilon x^T(t)Px(t) \leq 0. \tag{2.54}$$

Since $V(t) \geq 0$ while $\dot{V}(t) \leq 0$, this implies that $x(t)$ and $\Psi(t)\xi(t)$ converge to 0. Also, as $x(t) \to 0$, using (2.51) I have

$$\frac{d\Psi(t)}{dt} = -\hat{D}\Psi(t)\xi(t)\xi^T(t). \tag{2.55}$$

As a result, if $\xi(t)$ satisfies the PE condition, then the above differential equation is globally exponentially stable, $\Psi(t) \to 0$, based on the PE and Exponential Stability Theorem in [34], see Appendix. $\diamond$

As a result, I state the following theorem:

Theorem 2.1: Note that

$$\begin{aligned} Q_\Theta(s) &= (I - G(t)(sI - A_f)^{-1}B_f)^{-1} \\ &\quad \cdot(H(t) + F(t)(sI - A_f)^{-1}B_f). \end{aligned} \tag{2.56}$$

Under any stable transfer matrix $P(s)$ with minimum phase and known upper bound of the relative degree ($\mu := max(\mu_1, \cdots, \mu_m)$), the MIMO-FEL scheme and its learning law are stable if K_{fb} is chosen such that $M(s)$ in (2.49) is strictly positive real. Also, it yields $e(t) \to 0$, and $Q_\Theta(s)r(t) \to Q(s)r(t)$ if $\xi(t)$ satisfies the PE condition.

Proof: Based on lemma 2, if K_{fb} is chosen such that $M(s)$ is strictly positive real, then the solution of the adaptive law based on FEL will tend to a constant matrix which proves the stability. Also, if $\xi(t)$ satisfies the PE condition, then $\Psi(t) \to 0$ and $e(t) \to 0$, which result in $u_{ff} \to u_0$ as time proceeds. $\diamond$

The above result is an extension of [9] to MIMO systems. A possible drawback in this approach is that it requires the positive realness condition, which is not always guaranteed.

Following [10], I establish the stability of the FEL without the positive realness condition in the case of biproper systems *i.e., ($W(s) = I$).* The idea is how to make $M(s)$ equal to the identity matrix by influencing the kind of dynamics which is based on the error signal. The new vector $\xi_e(t)$ is generated by $e(t)$ with the following dynamics

$$\dot{\xi}_e(t) = A_f \xi_e(t) + B_f e(t), \tag{2.57}$$

where A_f and B_f are as defined before.

From (2.46), and taking the Laplace transformation of (2.57), I have the following relation

$$[H_0 + F_0(sI - A_f)^{-1}B_f - K_{fb}]e(t) = [H_0 - K_{fb}]e(t) + F_0\xi_e(t). \tag{2.58}$$

Then, substituting (2.58) into (2.45), and defining

$$\zeta := \begin{bmatrix} \xi_1(t) + \xi_e(t) \\ \xi_2(t) \\ r(t) \end{bmatrix}, \tag{2.59}$$

I obtain

$$\hat{u}_0(t) = \Theta_0 \zeta(t), \tag{2.60}$$

which is linearly parameterized by Θ_0. To derive an adaptive law for the FEL, I define $\hat{u}(t)$ by replacing Θ_0 in (2.60) with $\Theta(t)$

$$\hat{u}(t) = \Theta(t)\zeta(t). \tag{2.61}$$

I now define an error signal $s(t)$ as

$$s(t) := \hat{u_0}(t) - \hat{u}(t) = -\Psi(t)\zeta(t). \tag{2.62}$$

Using the gradient method as before with error function $s(t)$, the learning law becomes

$$\frac{d\Theta(t)}{dt} = \alpha s(t)\zeta^T(t). \tag{2.63}$$

Using (2.43) and (2.62), the learning law can be written as

$$\frac{d\Psi(t)}{dt} = -\alpha\Psi(t)\zeta(t)\zeta^T(t). \tag{2.64}$$

The stability of the above differential equation (2.64) can be verified using the following lemma.

Lemma 2.3: Let $\zeta(t)$ be an arbitrary time-varying vector. Then, the solution of the differential equation (2.64) tends to a constant matrix Ψ_0 such that $\Psi_0\zeta(t) \to 0$. If $\zeta(t)$ satisfies the PE condition, then the above matrix Ψ_0 is equal to zero matrix.

Proof: Let the Lyapunov function be defined as follows:

$$V(t) = \frac{1}{2}tr[\Psi^T(t)\Psi(t)]. \tag{2.65}$$

Then, the derivative along the trajectory is given by

$$\begin{aligned} \dot{V}(t) &= tr[\Psi^T(t)\dot{\Psi}(t)] \\ &= -tr[\Psi^T(t)\Psi(t)\zeta(t)\zeta^T(t)] \leq 0. \end{aligned} \tag{2.66}$$

Since $V(t) \geq 0$ while $\dot{V}(t) \leq 0$, this implies that the above differential equation is stable according to the Lyapunov theorem. Therefore, $\Psi(t)\zeta(t)$ converges to zero vector and thus $\frac{d\Psi(t)}{dt} \to 0$. For the second part of lemma 2, if $\zeta(t)$ is PE then the learning law is globally exponentially stable, namely $\Psi(t) \to 0$ based on the PE and Exponential Stability Theorem [34]. $\diamond$

Thus I derive the following theorem:

Theorem 2.2: Under any stable transfer matrix $P(s)$ with minimum phase and known upper bound of the relative degree ($\mu := max(\mu_1, \cdots, \mu_m)$), the MIMO-FEL scheme and its learning law (2.64) are stable. Also, it yields $e(t) \to 0$ if $\zeta(t)$ satisfies the PE condition.

Proof: From lemma 3, $\Psi(t)$ tends to a constant matrix Ψ_0 such that $\Psi_0\zeta(t) \to 0$. Therefore, $s(t) = -\Psi(t)\zeta(t) \to 0$. If $\zeta(t)$ satisfies the PE condition then Ψ_0 is equal to **0**, and then $\Theta(t) \to \Theta_0$ using (2.43). This implies that $e(t) \to 0$ as time proceeds. ⋄

Corollary: When $e(t) \to 0$ as $t \to \infty$, then

$$\Theta(t) \to \Theta_0, \ and \ Q_\Theta(s)r(t) \to Q(s)r(t). \tag{2.67}$$

The corollary concludes that the adaptive feedforward control based on MIMO-FEL will match the exact one, which is based on the interactorization concept as time proceeds.

2.5. Simulation Results

To show the effectiveness of the proposed method and summarize the procedure, I perform numerical simulation. First, I assume that A, B, C are known so that I can calculate the exact feedforward controller $Q(s)$. Then, I will use the learning law to adjust the tunable $\Theta(t)$ and compare it with a nominal one.

Giving

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & -4 & 1 \\ 3 & 1 & -5 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix},$$

I have the following transfer matrix of the plant:

$$P(s) = \begin{bmatrix} \frac{s^2+4s+1}{s^3+8s^2+19s+23} & \frac{-2s-19}{s^3+8s^2+19s+23} \\ \frac{2s^2+9s+7}{s^3+8s^2+19s+23} & \frac{s^2+5s-18}{s^3+8s^2+19s+23} \end{bmatrix}.$$

Using (2.3), I have $\mu_1 = 1$, $\mu_2 = 1$, and

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix},$$

which is invertible. Then, I select the interactor

$$L(s) = \begin{bmatrix} s+2 & 0 \\ 0 & s+2.5 \end{bmatrix}.$$

I now calculate the gain R to find the exact $Q(s)$ using equations (2.9) and (2.13):

$$R = \begin{bmatrix} 5 & -2 & -2 \\ -3 & 2 & 3 \end{bmatrix},$$

$$Q(s) = \begin{bmatrix} \frac{s^2+5s-18}{s^2+7s+10} & \frac{2s+19}{s^2+7.5s+12.5} \\ \frac{-2s^2-9s-7}{s^2+7s+10} & \frac{s^2+4s+1}{s^2+7.5s+12.5} \end{bmatrix}. \tag{2.68}$$

It can be verified that $P(s)Q(s) = W(s) = L^{-1}(s)$.

Second, I tune $\Theta(t)$ using the learning rule (2.38) so that I obtain the inverse of the system provided that the reference input is a sinusoidal signal ($r(t) = [\sin(t), \sin(t)]^T$). Thus, I need to set stable and controllable A_f and B_f based on the upper bound of the relative degree (2.11) $\mu = 1$, (see (2.18) and (2.19)),

$$A_f = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}, B_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

I choose the following feedback gain matrix that maintains the closed-loop stability and satisfies the SPR condition

$$K_{fb} = \begin{bmatrix} 0.1 & 0.25 \\ -0.3 & -0.15 \end{bmatrix}.$$

The simulation results in Figs. 2.2 and 2.3, using only a feedback controller, show bad tracking performance. However, it is shown in Figs. 2.4 and 2.5 that all outputs $y(t)$ track their reference inputs $w(t)$ using the proposed MIMO-FEL. Also, it is shown in Fig. 2.6 that the error signals tend to zero for MIMO-FEL performance. Furthermore, all the parameters $F(t)$, $G(t)$ and $H(t)$ which correspond to $\Theta(t)$ converge to their nominal values since $e(t) \to 0$, as shown in Fig. 2.7. The resulting matrices are as follows:

$$F_0 = \begin{bmatrix} 0.5888 & 0.1636 \\ -0.0856 & 0.2545 \end{bmatrix},$$

$$G_0 = \begin{bmatrix} 0.6524 & -0.0452 \\ -0.1542 & 0.1728 \end{bmatrix},$$

$$H_0 = \begin{bmatrix} -0.0032 & 2.5197 \\ -1.0685 & -0.0875 \end{bmatrix}.$$

By substituting the above matrices and the defined A_f and B_f, the estimated transfer matrix can be obtained, as follows:

$$\begin{aligned} Q_\Theta(s) &= [I - G_0(sI - A_f)^{-1}B_f]^{-1}[H_0 + F_0(sI - A_f)^{-1})B_f] \\ &= \begin{bmatrix} \frac{0.4188s^2+4.314s+10.39}{s^2+6.721s+11.23} & \frac{2.394s^2+18.4s+35.12}{s^2+6.721s+11.23} \\ \frac{-1.013s^2-7.22s-12.59}{s^2+6.721s+11.23} & \frac{-0.0455s^2+0.001115s+0.5082}{s^2+6.721s+11.23} \end{bmatrix}. \end{aligned} \tag{2.69}$$

It should be noted that the exact $Q(s)$ (2.68) is the true inverse of the plant, while the $Q_\Theta(s)$ (2.69) is an estimated inverse for the specified reference inputs $r(t)$. This means that for a different $r(t)$, I will obtain a different estimated inverse. It seems that this is because of a lack of PE condition in this case. In Figs. 2.8 and 2.9, I compare the outputs of the true and estimated inverse against $r(t)$:

$u^*_{ff} = Q(s)r,$

$u_{ff} = Q_\Theta(s)r.$

It is shown from the above that the adaptive feedforward inputs approach to the exact one which clarify the obtained corollary. This shows that the adaptive rule successfully learns the estimated inverse of the MIMO system, the main goal of this chapter.

2.6. Conclusion

In this chapter, learning control structures have been proposed for MIMO systems using FEL. By using linear system parameterization as a function approximator of the feedforward control, a learning law has been derived to adjust the inverse of the plant. A theoretical treatment of how to generalize FEL to MIMO systems has been studied in the framework of adaptive control. The FEL scheme was generalized to MIMO systems which are not necessarily biproper, and hence not invertible with properness. The feedforward controller is not designed on the basis of the process model, but is trained on-line during control using the feedback error signal as a learning signal.

Moreover, the exact inverse of the plant has been derived theoretically using the inverse interactorization concept, and can be used as a guide to check the correctness of the learning control. Furthermore, the stability of the tuning rule has been proved with and without positive realness condition.

The simulation results have shown that MIMO-FEL scheme can improve the system performance drastically.

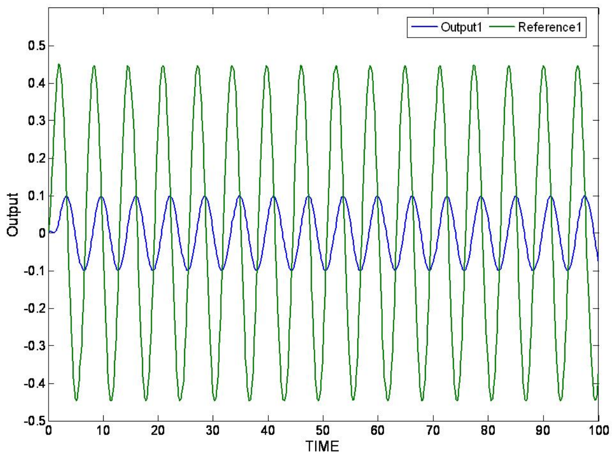

Figure 2.2. Reference vs. Output without MIMO-FEL:Channel 1

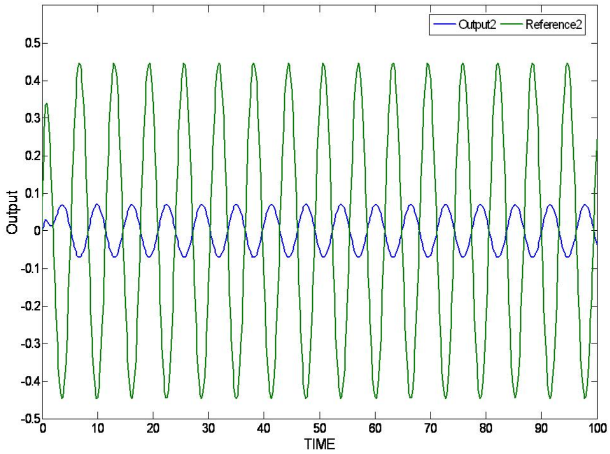

Figure 2.3. Reference vs. Output without MIMO-FEL:Channel 2

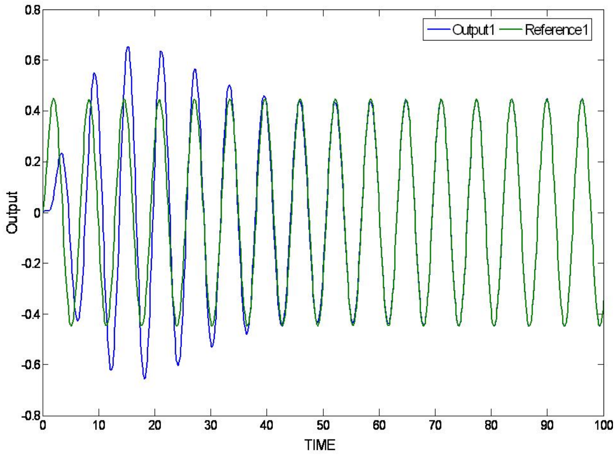

Figure 2.4. Reference vs. Output using MIMO-FEL: Channel 1

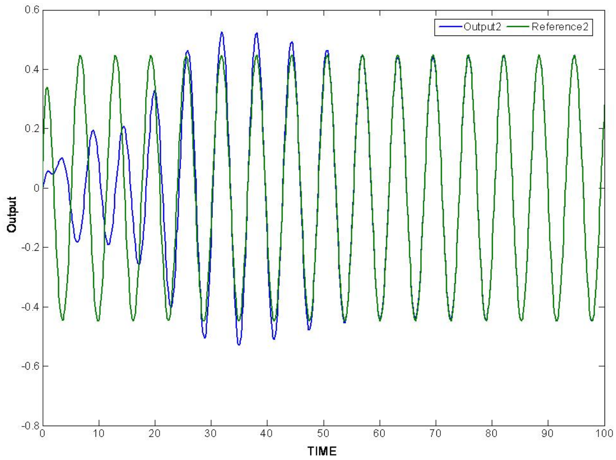

Figure 2.5. Reference vs. Output using MIMO-FEL: Channel 2

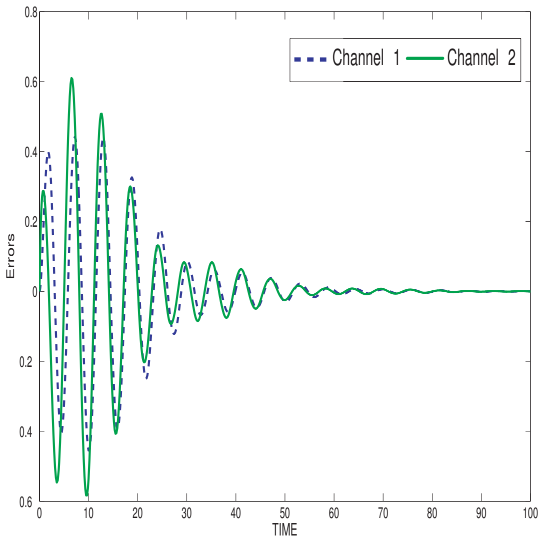

Figure 2.6. Tracking Errors for MIMO-FEL

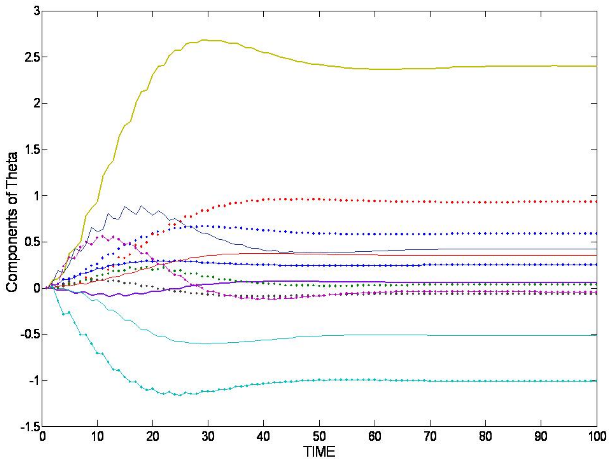

Figure 2.7. Time Evolution of $\Theta(t)$

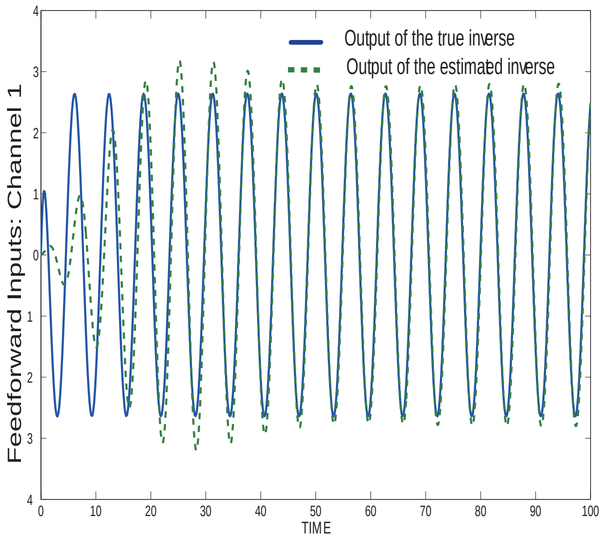

Figure 2.8. Feedforward Inputs u_{ff}, u^*_{ff}: Channel 1

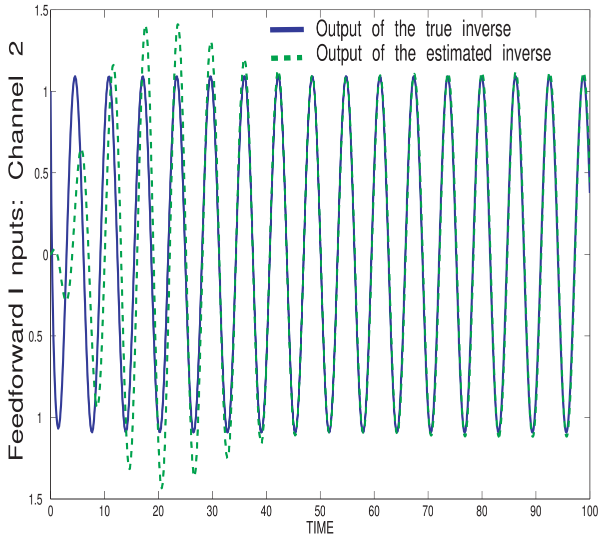

Figure 2.9. Feedforward Inputs u_{ff}, u^*_{ff}: Channel 2

Chapter 3

Closed-loop Identification Based on MIMO-FEL

Motivated by the recent development of FEL, this chapter proposes a method for closed-loop identification of a MIMO plant. Given a roughly designed control system, a feedforward controller is constructed by learning to achieve desirable responses. The trained feedforward controller then gives a model of the plant, which is effective for re-designing the control system to improve performance. The effectiveness of the method is verified through numerical simulation.

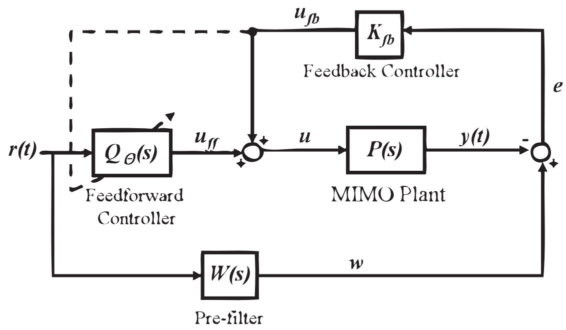

Figure 3.1. MIMO-FEL Identification Scheme

3.1. Introduction

In FEL, the parameter in the feedforward controller is tuned so that the feedback error converges to zero. An interesting point is that, after convergence, the acquired feedforward controller has sufficient knowledge about the inverse model of the plant, since its cascade connection with the plant is equal to the pre-filter; see Fig. 3.1. Thus, it is natural to expect that I can identify the plant parameter from this knowledge. In other words, closed-loop identification is achieved by applying FEL to the plant. The obtained system parameters can be used for re-designing the control system to improve performance.

In this chapter, I first use the acquired feedforward controller based on MIMO-FEL to obtain the system model. The performance of system identification is then evaluated with some examples in various conditions.

Closed-loop identification is attractive since it gives a model during operation (*i.e.*, on-line), but this is a challenging task due to numerical difficulty. This is the case also in the proposed scheme, but the result emerges to be adequate. A striking feature is that even an unstable plant can be identified provided that the closed-loop is stable.

3.2. MIMO-FEL Identification Analysis

The main objective of FEL, as mentioned above, is to improve the tracking performance by means of learning. It is then natural to expect that the acquired feedforward controller $Q_\Theta(s)$ has sufficient knowledge about the plant model, and hence I can identify the plant parameter from this knowledge.

Recall that if I take

$$u_0(t) = Q(s)r(t), \tag{3.1}$$

then I readily have $e \equiv 0$ in Fig. 3.1. Hence, if $Q_\Theta(s)$ converges to $Q(s)$ in FEL, then the control objective is achieved, but *not necessarily* vice versa. I will show this below.

I note that the parameterization of (2.24) can yield an arbitrary transfer matrix from $r(t)$ to $u_{\rm ff}(t)$. By taking the Laplace transformation of (2.24), I have

$$\begin{aligned} u_{\rm ff}(t) &= F(t)(sI - A_f)^{-1}B_f r(t) + G(t)(sI - A_f)^{-1}B_f u_{\rm ff}(t) + H(t)r(t) \\ &= \Big[I - G(t)(sI - A_f)^{-1}B_f\Big]^{-1} \cdot \Big\{H(t) + F(t)(sI - A_f)^{-1}B_f\Big\} r(t) \\ &=: Q_\Theta(s)r(t). \end{aligned} \tag{3.2}$$

It is already known that $e(t) \to 0$ as $t \to \infty$ by the learning law of Section 2.3. Then I have

$$P(s)Q_\Theta(s)r(t) = W(s)r(t), \tag{3.3}$$

at least for the given $r(t)$. Further, if $P(s)Q_\Theta(s) = W(s)$, then the plant parameter can be identified based on this learning feedforward controller. To guarantee this, I need to take $r(t)$ sufficiently rich (*i.e.*, satisfying the PE condition).

If I do not take sufficiently rich $r(t)$ as a reference signal, correct identification is no longer expected, although FEL nonetheless assures the perfect tracking performance. Even so, I may still be able to estimate the frequency characteristic of $P(s)$ to some extent, because good tracking is achieved for $r(t)$ with limited frequency components. This is an interesting issue, whose further analysis will be discussed in Chapter 4.

Thus, the procedure of closed-loop identification based on MIMO-FEL can be summarized as follows:

Step 1: I use the learning law (2.38) to tune the linear filter parameter in (2.24).

Step 2: As $\Theta(t)$ converges, I use the parameter to estimate the plant parameter by $P(s) = W(s)Q_{\Theta}^{-1}(s)$. The inversion is feasible by (3.2).

Step 3: If $r(t)$ is sufficiently rich, then I obtain the true value. Otherwise, I only obtain partial knowledge of $P(s)$.

3.3. Simulation Results

I consider two examples with first and third order systems. First, to illustrate the idea of the closed-loop identification, I perform numerical simulation using a simplest first order system.

Given

$$A = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.5 & 1 \\ 1.5 & 2 \end{bmatrix},$$

I have the following transfer matrix of the plant:

$$P(s) = \begin{bmatrix} \frac{1}{s+5} & \frac{2}{s+5} \\ \frac{3}{s+5} & \frac{4}{s+5} \end{bmatrix}.$$

Using (2.3), I have $\mu_1 = 1$, $\mu_2 = 1$, and

$$\Lambda = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},$$

which is invertible. Then, I select the following interactor

$$L(s) = \begin{bmatrix} s+1 & 0 \\ 0 & s+1 \end{bmatrix}.$$

I now calculate the gain R to find the exact $Q(s)$, using equations (2.9) and (2.13):

$$R = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix},$$

$$Q(s) = \begin{bmatrix} \frac{-2s-10}{s+1} & \frac{s+5}{s+1} \\ \frac{1.5s+7.5}{s+1} & \frac{-0.5s-2.5}{s+1} \end{bmatrix}.$$

It can be verified that $P(s)Q(s) = W(s) = L^{-1}(s)$.

Second, without using A, B, C, or R, I tune $\Theta(t)$ using the learning rule (2.38) so that I obtain the inverse of the system. In order that the reference input is sufficiently rich, I take $r(t)$ as a pseudo-random binary signal (PRBS) generated for identification purposes in MATLAB, as shown in Fig. 3.2. Thus, I need to set A_f and B_f based on the upper bound of the relative degree (2.11) $\mu = 1$ (see (2.18) and (2.19)) as

$$A_f = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}, B_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

I choose the following feedback gain matrix that maintains the closed-loop stability and satisfy the SPR condition

$$K_{fb} = \begin{bmatrix} 6 & 4 \\ 7 & 3 \end{bmatrix}.$$

The time evolution of $\Theta(t)$ can be seen in Fig. 3.3. The convergent value is

$$\Theta = \begin{bmatrix} -1.9929 & 0.9905 & 3.0039 & 0.0042 & -2.0006 & 1.0022 \\ 1.4932 & -0.4994 & 0.0016 & 3.0032 & 1.5016 & -0.4997 \end{bmatrix}.$$

The nominal Θ_0 is

$$\Theta_0 = \begin{bmatrix} -2 & 1 & 3 & 0 & -2 & 1 \\ 1.5 & -0.5 & 0 & 3 & 1.5 & -0.5 \end{bmatrix}.$$

It can be seen clearly that $\Theta(t) \to \Theta_0$. I can then obtain the learning feedforward controller $Q_{\Theta_0}(s)$ from (3.2)

$$Q_{\Theta_0}(s) = \begin{bmatrix} \frac{-2.001s^2-11.98s-9.932}{s^2+1.993s+0.9929} & \frac{1.002s^2+5.996s+4.973}{s^2+1.993s+0.9929} \\ \frac{1.502s^2+8.992s+7.454}{s^2+1.993s+0.9929} & \frac{-0.4997s^2-2.994s-2.48}{s^2+1.993s+0.9929} \end{bmatrix}.$$

Note that if I round off the convergent Θ, then I have $\Theta(t) = \Theta_0$ as $t \to \infty$. As a result, if I substitute Θ_0 in (3.2), I have $Q_{\Theta_0}(s) = Q(s)$.

I now estimate the plant using $Q_\Theta(s)$ in (3.2)

$$\hat{P}(s) = \begin{bmatrix} \frac{0.9891s^2+5.927s+4.91}{s^3+10.96s^2+34.74s+24.79} & \frac{1.984s^2+11.87s+9.843}{s^3+10.96s^2+34.74s+24.79} \\ \frac{2.972s^2+17.8s+14.76}{s^3+10.96s^2+34.74s+24.79} & \frac{3.96s^2+23.72s+19.66}{s^3+10.96s^2+34.74s+24.79} \end{bmatrix}.$$

The comparison between the estimated plant $\hat{P}(s)$ and the true plant $P(s)$ is shown in Figs. 3.4 and 3.5 in terms of the bode plot and step response. The estimated plant matches the true one. Further, if I round off the convergent Θ, I then obtain $\hat{P}(s) = P(s)$.

To check the dependency on signals, I also tested a reference signal which is not sufficiently rich ($r_c(t) = [\sin(t)\ \ \cos(t)]^T$). The time evolution of $\Theta_c(t)$ can be seen in Fig. 3.6. The convergent value is

$$\Theta_c = \begin{bmatrix} -0.6448 & 0.1530 & 1.0810 & -1.0344 & -2.4260 & 1.3156 \\ 0.5205 & -0.0527 & -0.6753 & 0.7263 & 2.0291 & -0.6936 \end{bmatrix},$$

which is far from the nominal value Θ_0. However, I have $e(t) \to 0$ as shown in Fig. 3.7. The obtained feedforward learning controller is:

$$Q_{\Theta_c}(s) = \begin{bmatrix} \frac{-2.426s^2-20.39s-42.81}{s^2+6.193s+8.857} & \frac{1.316s^2+10.44s+20.65}{s^2+6.193s+8.857} \\ \frac{2.029s^2+16.2s+32.2}{s^2+6.193s+8.857} & \frac{-0.6936s^2-5.741s-11.91}{s^2+6.193s+8.857} \end{bmatrix}.$$

The second example (*i.e.*, third order system) is

$$P(s) = \begin{bmatrix} \frac{s^2+4s+1}{s^3+8s^2+19s+23} & 0 \\ 0 & \frac{2s^2+9s+7}{s^3+8s^2+19s+23} \end{bmatrix}.$$

Using a sufficiently rich reference input $r(t)$ as in Fig. 3.2, $\Theta(t)$ has converged to the following matrix

$$\Theta = \begin{bmatrix} -0.9358 & -0.0029 & 3.8129 & -0.0181 & 0.7893 & -0.0007 \\ 0.0006 & -1.0457 & -0.0081 & 3.4219 & 0.0150 & 0.4516 \end{bmatrix}.$$

The estimated plant is as follows:

$$\hat{P}(s) = \begin{bmatrix} \frac{1.267s^2+2.371s+0.3993}{s^3+6.499s^2+13.74s+9.482} & \frac{0.001934s^2+0.0399s+0.04784}{s^3+6.499s^2+13.74s+9.482} \\ \frac{-0.04221s^2-0.1604s+0.01846}{s^3+6.999s^2+15.99s+11.85} & \frac{2.214s^2+7.512s+3.6}{s^3+6.999s^2+15.99s+11.85} \end{bmatrix}.$$

The comparison between the estimated plant $\hat{P}(s)$ and the true plant $P(s)$ is shown in Figs. 3.8 and 3.9 in terms of the bode plot and step response. In Fig. 3.8, the estimated plant matches the true one for the diagonal elements, while the off-diagonal elements are close to zero in terms of the gain (*i.e.*, ≈ -40 dB, -70 dB, respectively). This is a good approximation. Fig. 3.9 also shows that the estimated plant is close to the actual one.

3.4. Conclusion

In this chapter, I have proposed a closed-loop identification technique by means of MIMO-FEL. If the reference signal is PE, the method has emerged to be effective in simulation.

Thus, the control performance can be improved by re-designing the controller with the obtained plant model. At this moment, the convergence is slow, although this is often the case in adaptation or learning algorithms. This drawback will be overcome in the next chapter.

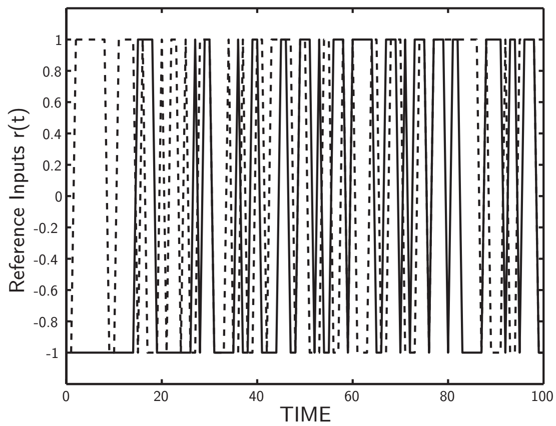

Figure 3.2. Reference Inputs: solid line: $r_1(t)$, dashed line: $r_2(t)$

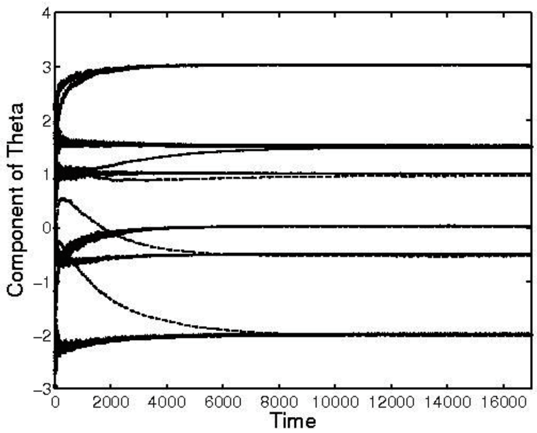

Figure 3.3. Time Evolution of $\Theta(t)$

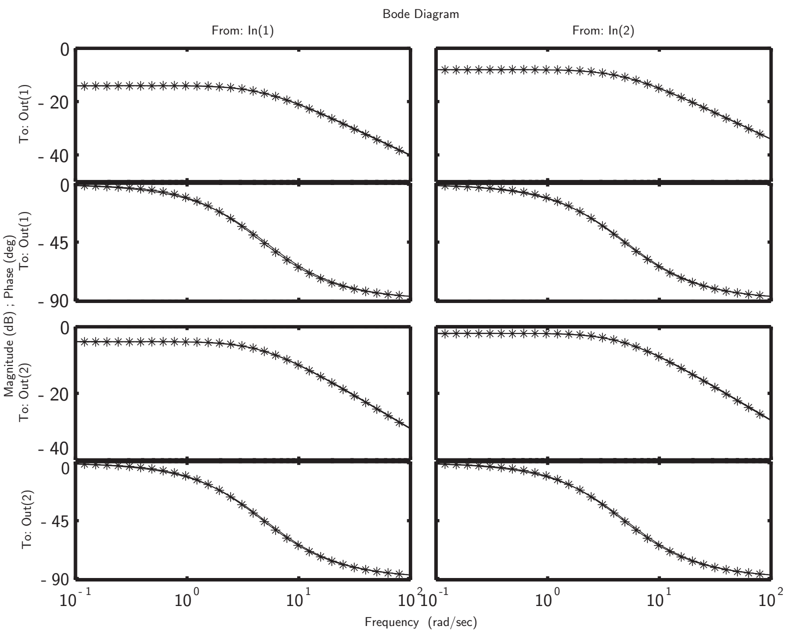

Figure 3.4. Bode Plot: solid line: true, star line: estimated

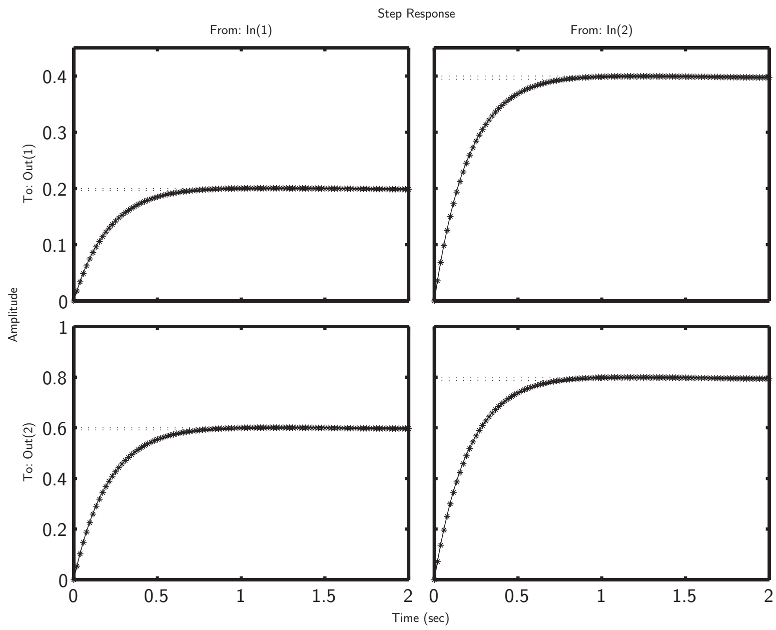

Figure 3.5. Step Response: solid line: true, star line: estimated

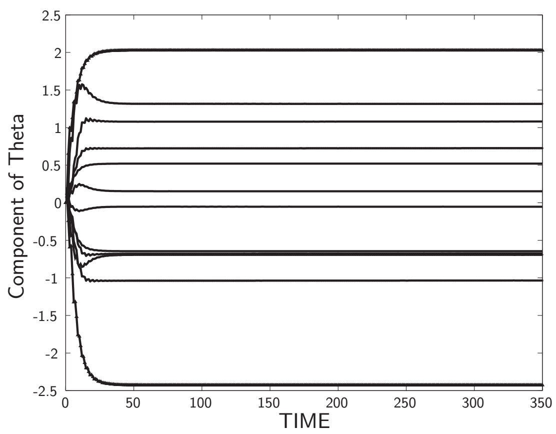

Figure 3.6. Time Evolution of $\Theta_c(t)$

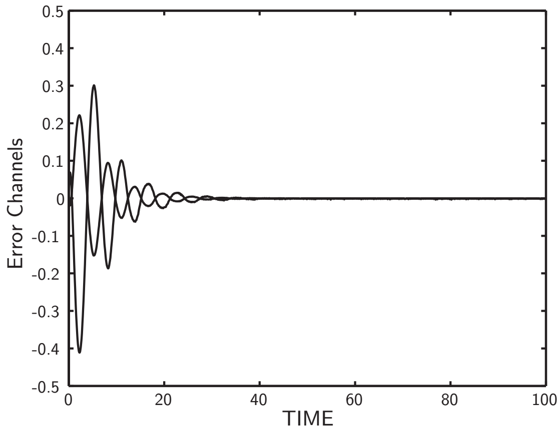

Figure 3.7. Error Channels: solid line: $e_1(t)$, dashed line: $e_2(t)$

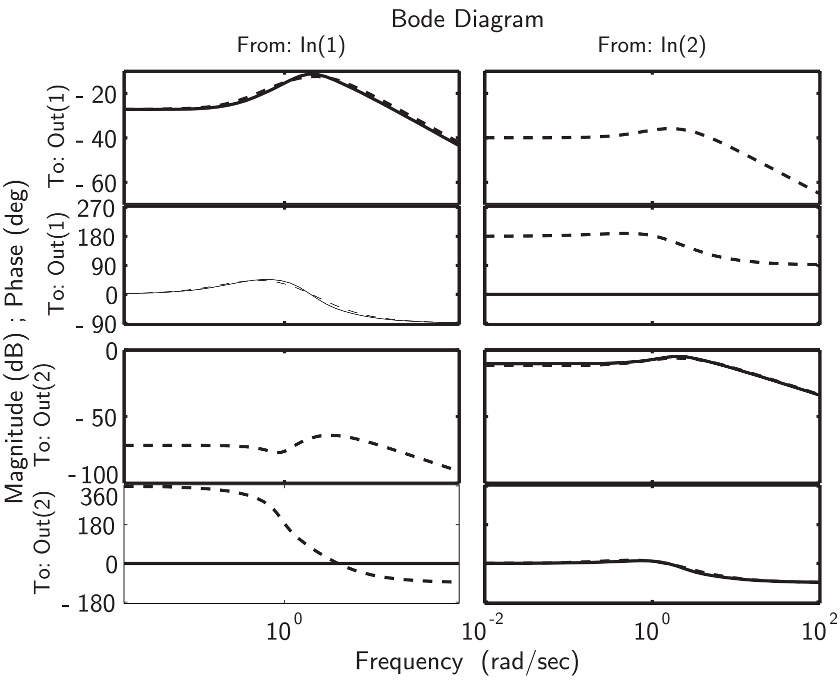

Figure 3.8. Bode Plot 2: solid line: true, dashed line: estimated

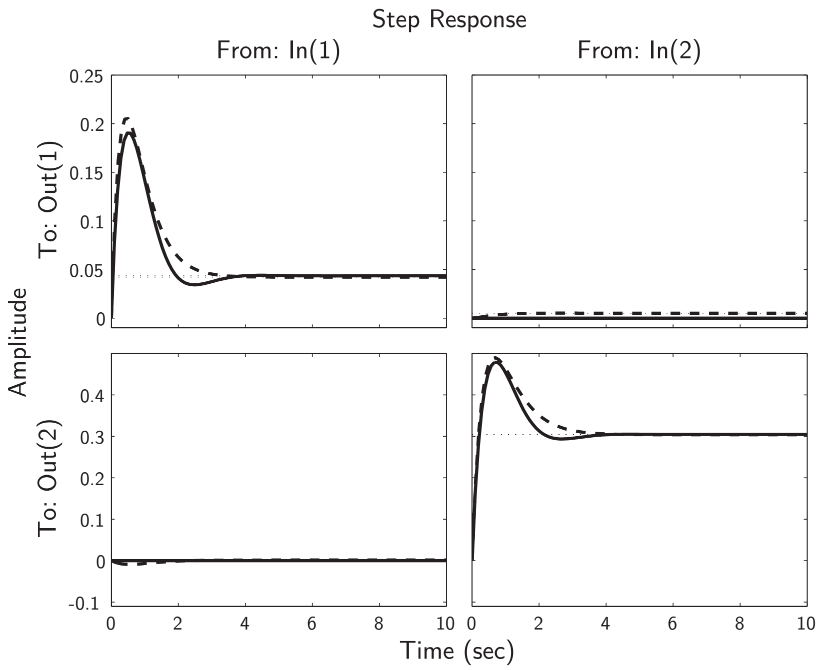

Figure 3.9. Step Response 2: solid line: true, dashed line: estimated

Chapter 4

FEL with Insufficient Excitation

This chapter studies the tracking error in a MIMO-FEL system having insufficient excitation. It is shown that the error converges to zero exponentially, even if the reference signal lacks the PE condition. Furthermore, by making full use of this fast convergence, the plant parameter is estimated under closed-loop operation based on frequency response. Simulation results show the effectiveness of the proposed method compared to a conventional approach.

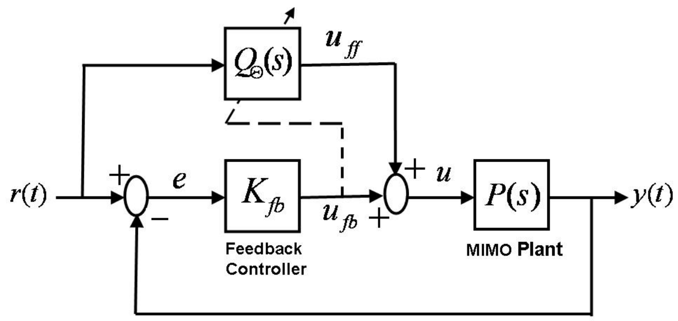

Figure 4.1. MIMO-FEL Architecture for the Biproper Case

4.1. Introduction

The objectives of this chapter are two-fold. First, I prove that the tracking error converges exponentially to zero even without the full PE condition; *i.e.*, even if signals are insufficiently rich. The merit of such a result is that, in practice, this condition is undesirable or even impossible to satisfy while good tracking performance is required.

Second, by making full use of this performance, the plant parameter is estimated under closed-loop operation. I apply a specific frequency of sinusoidal signal, which is a typical example of insufficient excitation, as a reference signal and make the feedforward controller learn to track this reference. After convergence, the trained controller reflects the inverse response of the plant at this frequency. I repeat this process for various frequencies. The plant parameter is then computed by solving a linear equation from these data.

It is widely known that closed-loop identification is difficult, since the input/output signals have correlation. The proposed method avoids such difficulty by canceling the feedback effect. As a result, the knowledge of feedback controller is not required, either. The rapid convergence of FEL achieves this process efficiently. The method does not follow conventional methodology where input/output data are given. Rather, it obtains the data by acting on the system with adaptive feedforward control.

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4.2. Exponential Tracking Error Convergence

The PE condition ensures parameter convergence as well as tracking error convergence for the FEL algorithm [9, 10, 36]. In this chapter, I relax this condition for the latter convergence [39].

First, recall the dynamics of the parameter estimation error defined by

$$\Psi(t) := \Theta_0 - \Theta(t). \tag{4.1}$$

By using (2.38), I have

$$\dot{\Psi}(t) = -\alpha u_{\text{fb}}(t)\xi^T(t). \tag{4.2}$$

In the scheme (see Fig. 4.1), I have

$$u_{\text{fb}} = u - u_{\text{ff}} = P^{-1}y - \hat{u}_o. \tag{4.3}$$

Furthermore,

$$P^{-1}y = Qy \cong Qr = u_o, \tag{4.4}$$

in the neighborhood of the exact parameter (i.e., when $\Psi \cong 0$). By using (4.1), (4.3), and (4.4) I obtain

$$u_{\text{fb}}(t) = u_0(t) - \hat{u}_0(t) = \Psi(t)\xi(t). \tag{4.5}$$

Substituting (4.5) in (4.2) I have

$$\dot{\Psi}(t) \;=\; -\alpha\Psi(t)\xi(t)\xi^T(t). \tag{4.6}$$

Thus, by defining the Lyapunov function $V = \frac{1}{2}tr\{\Psi^T\Psi\}$, I have its derivative

$$\begin{aligned}\dot{V}(t) &= tr[\Psi^T(t)\dot{\Psi}(t)] \\ &= -\alpha tr[\Psi^T(t)\Psi(t)\xi(t)\xi^T(t)] \leq 0.\end{aligned} \tag{4.7}$$

This proves that Ψ remains bounded. If ξ is bounded, then from (4.5) the error

$$e(t) = K_{\text{fb}}^{-1}u_{\text{fb}}(t) = K_{\text{fb}}^{-1}\Psi(t)\xi(t) \tag{4.8}$$

remains bounded. Furthermore, if $\dot{\xi}$ is bounded, then $\ddot{V}$ is also bounded and $\dot{V}$ is uniformly continuous. Thus Barbalat lemma [4], see Appendix, can be applied to ensure that $\lim_{t\to\infty} e(t) = 0$. However, this does not guarantee the exponential convergence of e to zero. An additional condition is usually imposed to this end [9, 10, 36].

In general, a vector signal, $\eta(t) = [\eta_1(t)\ \eta_2(t)\ \cdots \eta_p(t)]^T$, satisfies the PE condition, if there exists $\delta > 0$ such that

$$\Xi(t_0, \delta) = \int_{t_0}^{t_0+\delta} \eta(t)\eta^T(t)dt > 0, \tag{4.9}$$

where t_0 is the initial time. In what follows, I first show that if $\xi(t)$ in (2.25) is PE, then e converges to zero exponentially. Namely, there exist constants $\phi \geq 0$, $\sigma > 0$ such that

$$|e(t)| \leq \phi e^{-\sigma t}. \tag{4.10}$$

After that, I relax this sufficient condition.

Let $\mathbf{vec}(\Psi)$ denote the vector formed by stacking the columns of Ψ into one long vector:

$$\mathbf{vec}(\Psi) = (\psi_{11} \cdots \psi_{m1}\ \psi_{12} \cdots \cdots \psi_{m\ell})^T. \tag{4.11}$$

Then for any matrices X, Y and Z with appropriate dimensions, it is known that [40]

$$\mathbf{vec}(XYZ) = (Z^T \otimes X)\mathbf{vec}(Y), \tag{4.12}$$

where $\otimes$ is Kronecker product. As a result, (4.6) can be written as

$$\begin{aligned} \mathbf{vec}(\dot{\Psi}(t)) &= (\xi(t)\xi^T(t) \otimes (-\alpha I))\mathbf{vec}(\Psi(t)) \\ &= -\alpha \begin{bmatrix} \xi_1(t)I \\ \xi_2(t)I \\ \vdots \\ \xi_\ell(t)I \end{bmatrix} [\xi_1(t)I\ \xi_2(t)I\ \cdots\ \xi_\ell(t)I]\, \mathbf{vec}(\Psi(t)) \end{aligned} \tag{4.13}$$

This means that (4.6) is the system of linear differential equations. I now have the following lemma.

Lemma 4.1:

If $\xi(t)$ satisfies the PE condition, then

$$\xi(t)\xi^T(t) \otimes I = \begin{bmatrix} \xi_1(t)I \\ \xi_2(t)I \\ \vdots \\ \xi_\ell(t)I \end{bmatrix} [\xi_1(t)I \; \xi_2(t)I \; \cdots \; \xi_\ell(t)I] \tag{4.14}$$

also satisfies the PE condition.

Proof: In order to prove that

$$\int_{t_0}^{t_0+\delta} \xi(t)\xi^T(t) \otimes I dt =$$
$$\begin{bmatrix} \int_{t_0}^{t_0+\delta} \xi_1^2 dtI & \int_{t_0}^{t_0+\delta} \xi_1\xi_2 dtI & \cdots \\ \int_{t_0}^{t_0+\delta} \xi_2\xi_1 dtI & \int_{t_0}^{t_0+\delta} \xi_2^2 dtI & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} > 0,$$

it is enough to show that if

$$S = \begin{bmatrix} s_{11} & s_{12} & \cdots \\ s_{21} & s_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} > 0,$$

then

$$S \otimes I = \begin{bmatrix} s_{11}I & s_{12}I & \cdots \\ s_{21}I & s_{22}I & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} > 0.$$

There exists an orthogonal matrix V such that

$$V^T S V = \begin{bmatrix} \alpha_1 & & 0 \\ & \alpha_2 & \\ 0 & & \ddots \end{bmatrix}, \;\; \alpha_i > 0.$$

Hence, I obtain $(V \otimes I)^T S \otimes I (V \otimes I)$

$$= \begin{bmatrix} \alpha_1 & & 0 \\ & \alpha_2 & \\ 0 & & \ddots \end{bmatrix} \otimes I = \begin{bmatrix} \alpha_1 I & & 0 \\ & \alpha_2 I & \\ 0 & & \ddots \end{bmatrix} > 0. \; \diamond$$

I readily have the following theorem.

<u>Theorem 4.1:</u>

If $\xi(t)$ is PE, then (4.13) is globally exponentially stable, which implies that $e(t)$ also converges to zero exponentially.

Proof: The first statement holds by applying PE and Exponential Stability Theorem [34], see Appendix, to the vector differential equation (4.13), with Lemma 1. The second holds by (4.8). $\diamond$

I now proceed to the case where ξ is not fully excited, the first main objective of the paper. I start by defining the correlation matrix,

$$M = \int_{t_0}^{\infty} \xi(t)\xi^T(t)dt > 0. \tag{4.15}$$

The constant matrix M is positive definite if ξ is PE. If not, however, then M is positive semidefinite anyway, and there exists the eigenvalue decomposition:

$$M = R \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix} R^T, \ \Lambda = \text{diag}\{\lambda_1, \ldots, \lambda_p, \} \tag{4.16}$$

where $R^T = R^{-1}$ and $\lambda_1 \geq \cdots \geq \lambda_p > 0$. Below I will prove that the error system (4.5) and (4.6) can be written equivalently as a reduced system.

<u>Theorem 4.2:</u>

Using the MIMO-FEL adaptive law (2.38), the smallest nonzero eigenvalue λ_p always exists unless $\xi \equiv 0$, $e \equiv 0$. Further, the tracking error e converges to zero exponentially.

Proof: If $M = 0$ then, from (4.15) $\xi \equiv 0$, and from (4.8) the error $e \equiv 0$. Hence, $M \neq 0$ and $\lambda_p > 0$ exists. Defining

$$\rho(t) = R^T \xi(t), \ \ \Omega(t) = \Psi(t)R, \tag{4.17}$$

$$\rho(t) = \begin{bmatrix} \rho_1(t) \\ \rho_2(t) \end{bmatrix}, \ \ \Omega(t) = \begin{bmatrix} \Omega_2(t) & \Omega_2(t) \end{bmatrix}, \tag{4.18}$$

in block sizes compatible with (4.16), the error (4.5) can be written as follows:

$$u_{\text{fb}}(t) = \Psi R R^T \xi = \Omega \rho = \Omega_1 \rho_1 + \Omega_2 \rho_2. \tag{4.19}$$

From (4.16), the correlation matrix of ρ is computed as

$$\int_{t_0}^{\infty} \rho(t)\rho^T(t)dt = \begin{bmatrix} \Lambda & 0 \\ 0 & 0 \end{bmatrix}, \tag{4.20}$$

which implies from (4.18) that

$$\int_{t_0}^{\infty} \rho_1(t)\rho_1^T(t)dt = \Lambda > 0, \;\; \rho_2 \equiv 0. \tag{4.21}$$

Substituting (4.21) in (4.19) gives

$$u_{\text{fb}}(t) = \Omega_1(t)\rho_1(t). \tag{4.22}$$

This implies that the excitation of ρ has been reduced into the smaller vector ρ_1 which is persistently excited by (4.21).

A similar reduction can be shown for Ω as follows. By post-multiplying both sides of (4.6) by R,

$$\dot{\Psi}(t)R = -\alpha\Psi(t)\xi(t)\xi^T(t)R. \tag{4.23}$$

Using (4.17) and (4.5), I have

$$\dot{\Omega}(t) = -\alpha u_{\text{fb}}(t)\rho^T(t) = -\alpha\Omega_1(t)\rho_1(t)\rho^T(t) \tag{4.24}$$

by (4.22), which can be partitioned using (4.18) and (4.21) as

$$\dot{\Omega}_1(t) = -\alpha\Omega_1(t)\rho_1(t)\rho_1^T(t), \;\; \dot{\Omega}_2(t) = 0. \tag{4.25}$$

Since the reduced signal ρ_1 in (4.21) is persistently exciting, it follows that the reduced parameter error Ω_1 in (4.25) converges exponentially based on Theorem 1. As $\Omega_1 \to 0$, $u_{\text{fb}}(t) \to 0$ by (4.22), which ensures the exponential convergence of e by (4.8). $\diamond$

The proof concludes that there is no need for convergence of the full parameter matrix $\Theta(t)$ for the purpose of the exponential convergence of the tracking error. Rather, partial excitation of η, *i.e.*, ρ_1, is sufficient for the exponential convergence of e.

4.3. Parameter Estimation

The main advantage of FEL, as mentioned above, is the exponential convergence of the tracking error by means of learning. It is then natural to expect that the acquired $Q_\Theta(s)$ has some knowledge of the plant model. If, in particular, I apply a sinusoidal reference then $Q_\Theta(s)$ works as the inverse of the plant at this frequency. By testing various frequencies in this way, the plant parameter can be identified from such knowledge.

Namely, I can estimate plant parameter while in closed-loop operation. To be specific, I consider a left coprime factorization (LCF)

$$P(s) = \mathcal{D}^{-1}(s)\mathcal{N}(s), \tag{4.26}$$

where $\mathcal{D}(s)$ and $\mathcal{N}(s)$ are polynomial matrices defined by

$$\begin{array}{rcl} \mathcal{D}(s) & = & s^\mu I + s^{\mu-1}D_1 + \ldots + D_\mu, \\ \mathcal{N}(s) & = & s^\mu N_0 + s^{\mu-1}N_1 + \ldots + N_\mu, \end{array} \tag{4.27}$$

where μ is assumed to be known. N_0 is nonsingular because $P(s)$ is biproper. The second objective of the paper is to estimate the coefficient matrices in (4.27) based on MIMO-FEL. I now apply the reference signal of the form

$$r_i{}^k(t) = \gamma_i{}^k \sin(\omega_i t), \;\; k = 1, \cdots, m, \tag{4.28}$$

using the same frequency ω_i and linearly independent vectors $\gamma_i{}^1, \cdots, \gamma_i{}^m$ for $i = 1, \cdots, \mu$. I then make the feedforward controller learn to track such references. After convergence, the trained controllers satisfy

$$P(j\omega_i)Q_{\Theta_i{}^k}(j\omega_i)\gamma_i{}^k = \gamma_i{}^k. \tag{4.29}$$

Hence, from (4.26) I obtain

$$\mathcal{N}(j\omega_i)[\xi_i{}^1 \cdots \xi_i{}^m] = \mathcal{D}(j\omega_i)[\gamma_i{}^1 \cdots \gamma_i{}^m]. \tag{4.30}$$

where $\xi_i{}^k = Q_{\Theta_i{}^k}(j\omega_i)\gamma_i{}^k$. Equation (4.30) is a linear equation with respect to the coefficient matrices of (4.27). By testing (4.28) for various frequencies and solving (4.30) for $i = 1, \cdots, \mu$, I obtain those coefficient matrices.

Take the second order biprober SISO system for example. I then have four parameters to be estimated. I need to apply two different frequencies to be able to estimate the parameters. By taking ${\gamma_i}^k = 1$, I obtain ${\xi_i}^k = Q_{{\Theta_i}^k}(j\omega_i)$. Then (4.30) becomes

$$(-\omega_i^2 + j\omega_i n_1 + n_2)(\alpha_i + j\beta_i) = (-\omega_i^2 + j\omega_i d_1 + d_2), \tag{4.31}$$

where $Q_{\Theta_i}(j\omega_i) = \alpha_i + j\beta_i$. Comparing its real and imaginary parts, I have

$$\left.\begin{array}{l} -\omega_i^2\alpha_i + \alpha_i n_2 - \omega_i\beta_i n_1 = -\omega_i^2 + d_2 \\ -\omega_i^2\beta_i + \beta_i n_2 + \omega_i\alpha_i n_1 = \omega_i d_1 \end{array}\right\} i = 1,\ 2. \tag{4.32}$$

The plant parameter is then computed by solving the following linear equation:

$$\begin{bmatrix} \omega_1 & 0 & -\alpha_1\omega_1 & -\beta_1 \\ \omega_2 & 0 & -\alpha_2\omega_2 & -\beta_2 \\ 0 & 1 & \beta_1\omega_1 & -\alpha_1 \\ 0 & 1 & \beta_2\omega_2 & -\alpha_2 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} -\beta_1\omega_1^2 \\ -\beta_2\omega_2^2 \\ \omega_1^2 - \alpha_1\omega_1^2 \\ \omega_2^2 - \alpha_2\omega_2^2 \end{bmatrix}.$$

In general, the algorithm is summarized as follows:

1. Put $i = 1$.
2. Apply sinusoidal reference input ${r_i}^k(t)$ at particular frequency ω_i.
3. Use the learning law (2.38) to tune the linear filter parameter in (2.24).
4. As $e(t)$ converges to zero, obtain the value of $\Theta_i(t)$.
5. From $\Theta_i(t)$, compute $Q_{\Theta_i}^{-1}(j\omega_i)$.
6. Go back to Step 2 and apply different frequency $i := i + 1$ until $i = \mu$, otherwise go to Step 7.
7. Based on the obtained frequency response data, solve the linear system of equations (4.30).

Note that the proposed algorithm avoids the input/output correlation of the signals, which is common in closed-loop identification, by adjusting feedforward input signals to cancel the feedback effect. The rapid convergence of FEL achieves this process efficiently. Another advantage of the proposed method is that it does not require knowledge of the feedback controller.

4.4. Simulation Results

To illustrate the proposed method, I perform numerical simulation. Consider the plant:

$$P(s) = \frac{s^2 + 3s + 2}{s^2 + 7s + 12}$$

I apply the reference input $r_i(t) = \sin(\omega_i t)$, for $\omega_1 = 1$ rad/sec and $\omega_2 = 2$ rad/sec. I choose a suitable feedback controller gain $K_{fb} = 5$, which maintains the closed-loop stability.

4.4.1 Proposed Method

Based on the degree of the system, I set

$$A_f = \begin{bmatrix} 0 & 1 \\ -5 & -5 \end{bmatrix}, B_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

I then tune $\Theta(t)$ using the learning rule (2.38) so that I can improve the tracking performance. Fig. 4.2 shows that the error signals $e_i(t)$ for the two frequencies tend to zero very fast. Then, I obtain the value of $\Theta_i(t)$ at the time when $e_i(t)$ vanishes; $e_1(t)$ and $e_2(t)$ vanish around $t_1 = 20$ and $t_2 = 40$, respectively. The trained feedforward controllers for the two frequencies are as follows:

$$Q_{\Theta_1}(s) = \frac{3.421s^2 + 16.77s + 18.28}{s^2 + 5.428s + 1.232},$$

$$Q_{\Theta_2}(s) = \frac{1.66s^2 + 7.978s + 10.6}{s^2 + 2.666s + 0.3708}.$$

Note that the resulting $Q_{\Theta_i}(s)$ does not represent the inverse of the plant, *i.e.*, $P(s)Q_{\Theta_i}(s) \neq 1$. $\Theta(t)$ does not converge to its nominal value, but to some other constant. This is because $r_i(t)$ lacks full excitation or the PE condition; nonetheless, the tracking is successful, which is the first result of this paper. To confirm this, I compute the correlation matrix

$$M = \begin{bmatrix} 1.2161 & 0.012 & 3.8782 & 3.0599 & 4.9343 \\ 0.012 & 1.2022 & -2.9742 & 3.8524 & 6.0719 \\ 3.8782 & -2.9742 & 20.1335 & 0.1532 & 0.6543 \\ 3.0599 & 3.8524 & 0.1532 & 20.1224 & 31.5214 \\ 4.9343 & 6.0719 & 0.6543 & 31.5214 & 50.2183 \end{bmatrix}$$

and find that the rank of M is 4.

I now see that from these trained feedforward controllers, the plant parameter can be estimated. The Bode plot in Fig. 4.3 shows that the true versus the estimated gain and phase are equal at the particular frequencies; $P(j\omega_i) = Q_{\Theta_i}^{-1}(j\omega_i)$. From the frequency response data $Q_{\Theta_i}^{-1}(j\omega_i)$, I solve the linear system given in Section 4.4 to estimate the plant parameter. The result is as follows:

$$P_{FEL}(s) = \frac{s^2 + 2.8387s + 1.8611}{s^2 + 6.8567s + 11.1334}.$$

The estimated parameters are close enough to the true one, but they can be improved by testing further frequencies. The parameter estimation can also be improved if I allow more time until parameter convergence.

4.4.2 Conventional Method

I adopt an approach based on 1DOF *i.e.*, without feedforward controller in Fig. 4.1. The frequency response data from the reference input and the output of the plant are used to estimate the plant parameter. In this simulation, the spectrum analyzer in MATLAB is used to measure the frequency response data. The data is collected at the same time as in Section 4.5.1 for fair comparison between both methods. I also use the same feedback controller, reference input and frequencies. From the frequency response data, I solve the linear system as in Step 7 in the algorithm to estimate the closed-loop transfer function. Then, using feedback controller knowledge, the plant is estimated as follows:

$$P_{con.}(s) = \frac{s^2 + 3.405s + 2.5697}{s^2 + 7.15s + 14.51}.$$

This estimation is biased from the actual plant parameter. Note that the estimation is again much improved if I allow more time.

4.4.3 Results Comparison

It can be seen clearly that the estimation based on the FEL is better than the conventional approach, as shown in Figs. 4.4 and 4.5. The former is fairly close to the actual one. I conclude that the proposed algorithm gives a good approximation of the plant model faster than the conventional method. Another merit of FEL estimation over the conventional approach is that it does not require feedback controller knowledge.

4.5. Conclusion

The main objective of this work was to prove the exponential convergence of the tracking error without full excitation in FEL. The merit of this result is that in many applications good tracking performance is required, while it is not desirable or even impossible to satisfy PE condition. Furthermore, the parameter estimation using FEL showed better results than the conventional approach after error convergence without depending on knowledge of the feedback controller as required by most of the conventional methods.

Compared to the recent work by Kaneko *et al.* [41], where 2DOF is used as in the proposed approach for closed-loop identification based on fictitious reference iterative tuning (FRIT), which requires only one-shot experimental data to identify the plant parameter. However, the parameter tuning is done off-line using the collected data, while the proposed method here works on-operation.

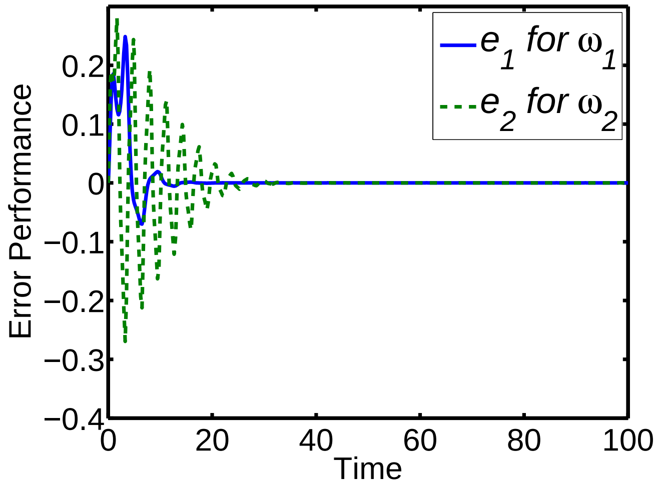


Figure 4.2. Tracking Errors

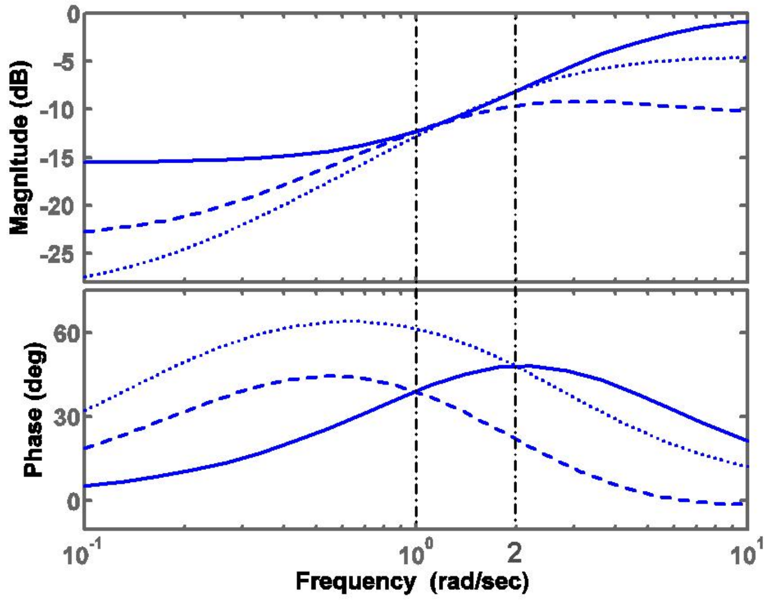


Figure 4.3. FEL Analysis Results: solid line: $P(s)$, dashed line: $Q_{\Theta_1}^{-1}(s)$, dotted line: $Q_{\Theta_2}^{-1}(s)$

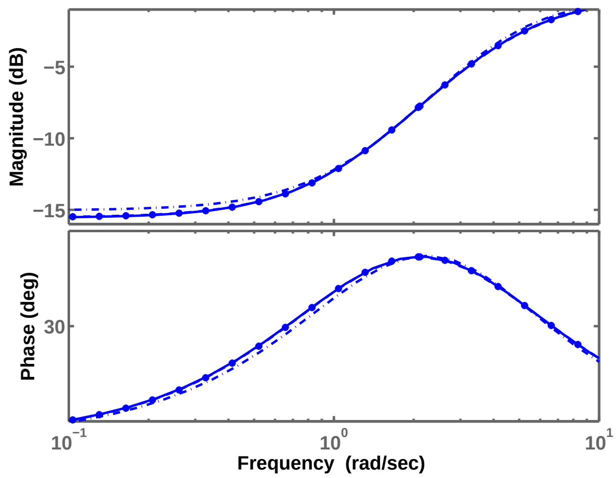


Figure 4.4. Comparison Results: solid line: $P(s)$, dashed line: $P_{FEL}(s)$, dotted line: $P_{con.}(s)$

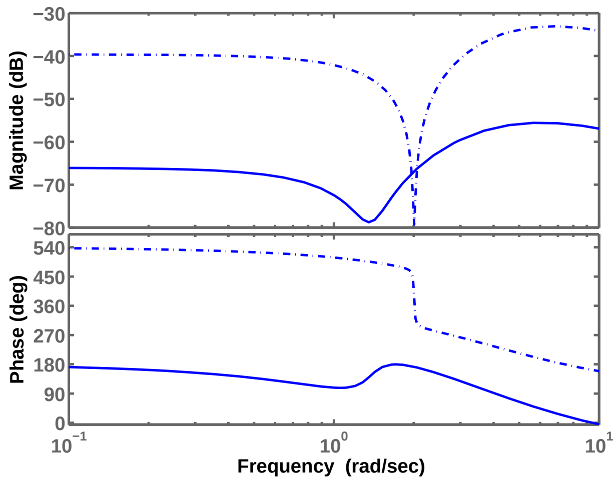


Figure 4.5. Comparison Difference: solid line: $P(s) - P_{FEL}(s)$, dash-dotted line: $P(s) - P_{con.}(s)$

Chapter 5

FEL for Writing One-stroke Characters by Two-Link Manipulator

In this chapter, a version of the problem of how to teach robots to write characters in an actual environment is considered. In particular, a feedforward controller is designed for two-link manipulators to improve tracking performance despite limited knowledge of the surroundings. An adaptive scheme, called MIMO-FEL, is employed to achieve the objective. After convergence, the feedforward controllers are switched depending on the target character to be written. The effectiveness of the proposed method is demonstrated with an experiment.

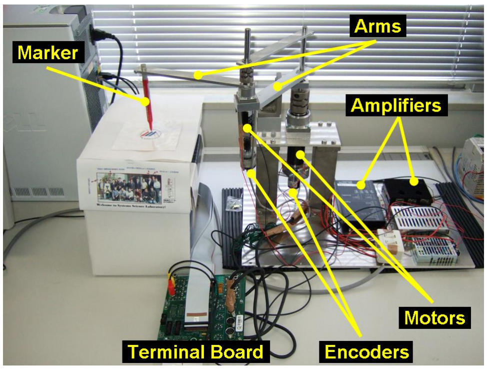


Figure 5.1. Experimental Hardware

5.1. Introduction

High tracking performance is one of the most important requirements for robotics applications. To design a model-based feedforward controller with good tracking performance, an accurate model of the process is needed. However, factors such as uncertainty, nonlinearity or time-varying behavior make modeling and identification more difficult or expensive. To overcome this challenge, several adaptive and learning control techniques have been proposed [5, 6, 7, 30, 43]. There are in general two distinct adaptive control approaches. The first approach is called indirect adaptive control because the adaptive laws provide explicit estimates of the dynamics of the model parameter, which is then used in controller design [7]. The second is called direct adaptive control, as the adaptive laws adjust the control gains directly without parameter estimation [6]. These approaches have also been used to adjust the feedforward controller to obtain an accurate inverse model of the plant as in [44]. Adaptive inversion, which first needs to estimate a plant model, is less sensitive to plant uncertainties and variations but also adjusts itself to plant parameter changes.

However, two powerful model-free learning control methods, iterative learning control (ILC) and feedback error learning (FEL), have attracted much attention in the last two decades (see, *e.g.*, [1, 2, 3, 8, 9, 10]). ILC deals with repeating tracking tasks in a finite time interval. Thus, it yields the desired input through the iteration of trials with the reset action of initial conditions. FEL, proposed by

Kawato *et al.* [8], achieves an inverse model of the plant without extensive modeling by utilizing the error signal during continuous-time closed-loop operation. The key point of FEL is to use a learning law which depends on the feedback error in order to tune the feedforward controller parameters.

Miyamura and Kimura [9] have established a control theoretical validity of the FEL method in the frame of adaptive control for the SISO case, proving its stability based on strictly positive realness, whereas Muramatsu and Watanabe [10] have relaxed the positive realness condition of FEL. Following [9], Alali *el al.* in [35, 36, 37] have studied some generalizations of the FEL scheme.

FEL has been implemented successfully using ANN in many systems such as industrial robot, humanoid robot, inverted pendulum and flexible link [25, 27, 26, 45]. The experimental results have shown the effectiveness of using FEL to improve tracking performance. The present work further studies an application of the MIMO-FEL technique developed in [36] to a practical problem in terms of a two-link manipulator. The basic idea of this work is to achieve an approximated inverse of the plant adaptively, using linear parameterization instead of ANN, to improve tracking performance for each specific desired trajectory and also the speed of parameter convergence by means of MIMO-FEL. This also contrasts with achieving an exact inverse via precise system identification, which requires a huge amount of data and richness of the excitation input. Thus, by FEL, one can obtain an inverse model for a specific reference signal with a limited amount of data and a limited range of frequency components. In practice, the feedforward controllers are switched depending on the target character to be written. This is a clear contrast with the precise identification approach, which uses a single general-purpose controller.

The basic assumption made in Chapter 2 to prove the convergence of the proposed algorithm is that the plant is linear. Since the two-link manipulator is a typical nonlinear system, I have to fill this gap. There are basically two sources of the nonlinearity: the first one is caused by the friction force generated by the reduction gear of the motor and the time-varying inertia. The second one is caused by the trigonometric dependency of the angle of the motors to the X-Y coordinates of the hand position. I overcome the first one by means of high-gain local feedback, the second by restricting the working area of the hand position to

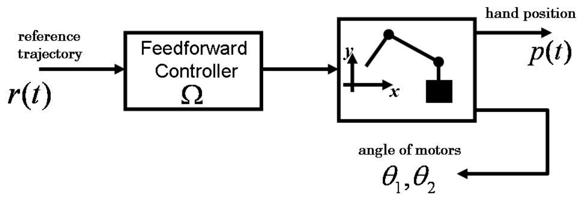


Figure 5.2. Feedforward Controller Design

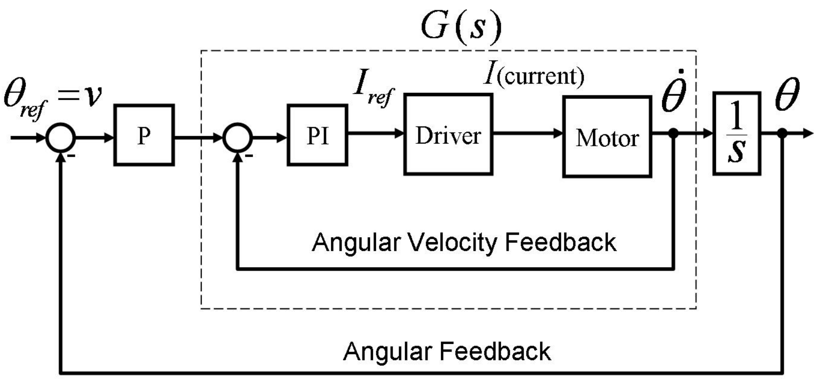


Figure 5.3. Local Feedback Structure

within a small neighborhood around the equilibrium point.

5.2. Dynamics of Two-Link Manipulator

I consider a two-link manipulator in Fig. 5.1. Each arm is driven by a DC motor with a reduction gear. The derivers apply the current to each motor in proportion to their input voltages. The objective here is to design a feedforward controller Ω for this manipulator system which achieves a good tracking performance, namely $\|p(t) - r(t)\| \to 0$ as $t \to \infty$ as illustrated in Fig. 5.2. As mentioned above, local angular velocity and angular feedback based on the encoder signal with relatively high gains are applied, as in Fig. 5.3. Thus, the nonlinearities due to friction or time-varying inertia are compensated. Assume that the behavior of the dashed line block in Fig. 5.3 is much improved by the inner PI loop, *i.e.*, $G(s) \simeq 1$. Then,

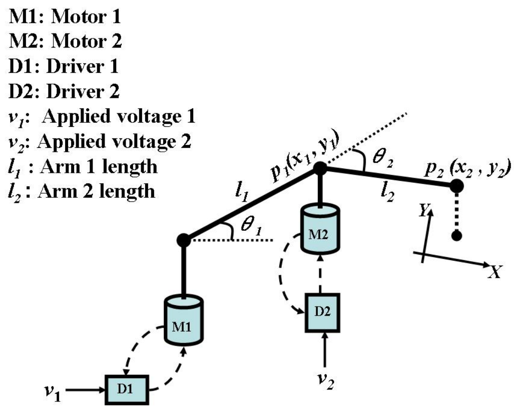


Figure 5.4. Two-Link Manipulator Model

the dynamics from input voltage v_i to angle θ_i can be approximated as

$$\theta_i = \frac{k_i}{\tau_i s + 1} v_i, \tag{5.1}$$

where τ_i and k_i are parameters that need to be identified. The x-y coordinates of the two points p_1 and p_2 in Fig. 5.4 are given as follows:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 \\ l_1 \sin\theta_1 \end{bmatrix}, \tag{5.2}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_2 - \theta_1) \\ l_1 \sin\theta_1 - l_2 \sin(\theta_2 - \theta_1) \end{bmatrix}. \tag{5.3}$$

Let

$$\begin{aligned} \theta_1 &= \theta_1^* + \Delta\theta_1, \\ \theta_2 &= \theta_2^* + \Delta\theta_2, \end{aligned} \tag{5.4}$$

where θ_1^* and θ_2^* are initial angles of the motor corresponding to the equilibrium point of the hand position. To overcome the nonlinearity effects caused by the trigonometric functions in (5.3), I restrict the working area of the hand position to within a small neighborhood around the equilibrium point. Then, the motion around (θ_1^*, θ_2^*) can be approximated by linear dynamics using Taylor series expansion. Therefore, the resulting hand position $p_2(x_2, y_2)$ is given as follows:

$$
\begin{aligned}
x_2 &= l_1 \cos\theta_1^* - l_1 \sin\theta_1^* \Delta\theta_1 + l_2 \cos(\theta_2^* - \theta_1^*) \\
&\quad - l_2 \sin(\theta_2^* - \theta_1^*)(\Delta\theta_2 - \Delta\theta_1) \\
&=: x_2^* + \{l_2 \sin(\theta_2^* - \theta_1^*) - l_1 \sin\theta_1^*\}\Delta\theta_1 - l_2 \sin(\theta_2^* - \theta_1^*)\Delta\theta_2 \\
y_2 &= l_1 \sin\theta_1^* + l_1 \cos\theta_1^* \Delta\theta_1 - l_2 \sin(\theta_2^* - \theta_1^*) \\
&\quad - l_2 \cos(\theta_2^* - \theta_1^*)(\Delta\theta_2 - \Delta\theta_1) \\
&=: y_2^* + \{l_2 \cos(\theta_2^* - \theta_1^*) - l_1 \cos\theta_1^*\}\Delta\theta_1 - l_2 \cos(\theta_2^* - \theta_1^*)\Delta\theta_2.
\end{aligned}
\tag{5.5}
$$

The coordinates x_2^* and y_2^* are given as

$$
\begin{aligned}
x_2^* &= l_1 \cos\theta_1^* + l_2 \cos(\theta_2^* - \theta_1^*), \\
y_2^* &= l_1 \sin\theta_1^* - l_2 \sin(\theta_2^* - \theta_1^*).
\end{aligned}
\tag{5.6}
$$

Note that (x_2^*, y_2^*) is the equilibrium point of linearization. Further, the following relationship is obtained from (5.1)

$$
\Delta\theta_i = \frac{k_i}{\tau_i s + 1}\Delta v_i,
\tag{5.7}
$$

where Δv_i denotes the deviation from the initial voltage. As a result, if I consider the motion in a small neighborhood of $p_2(x_2, y_2)$, then I can approximate the hand motion with the following linearized model as

$$
\begin{aligned}
\begin{bmatrix} \Delta x_2 \\ \Delta y_2 \end{bmatrix} &= M_1(\theta_1^*, \theta_2^*) M_2(s) \begin{bmatrix} \Delta v_1 \\ \Delta v_2 \end{bmatrix}, \\
&=: P(s) \begin{bmatrix} \Delta v_1 \\ \Delta v_2 \end{bmatrix},
\end{aligned}
\tag{5.8}
$$

where

$$
\begin{aligned}
\Delta x_2 &= x_2 - x_2^*, \\
\Delta y_2 &= y_2 - y_2^*,
\end{aligned}
\tag{5.9}
$$

$$
M_1(\theta_1^*, \theta_2^*) = \begin{bmatrix} l_2 \sin(\theta_2^* - \theta_1^*) - l_1 \sin\theta_1^* & -l_2 \sin(\theta_2^* - \theta_1^*) \\ l_2 \cos(\theta_2^* - \theta_1^*) + l_1 \cos\theta_1^* & -l_2 \cos(\theta_2^* - \theta_1^*) \end{bmatrix},
\tag{5.10}
$$

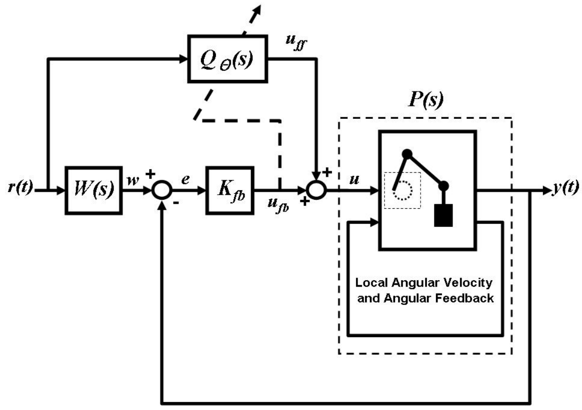


Figure 5.5. Two-Link Manipulator Control Scheme Using MIMO-FEL

$$M_2(s) = \begin{bmatrix} \frac{k_1}{\tau_1 s+1} & 0 \\ 0 & \frac{k_2}{\tau_2 s+1} \end{bmatrix}. \tag{5.11}$$

The theoretical contribution in Chapter 2 is two-fold: extension to MIMO and to strictly proper systems. The target system (5.8) is exactly what should be considered in the framework and I may apply the linear MIMO-FEL technique to this plant.

5.3. Controller Design

The two-link manipulator control scheme using MIMO-FEL is shown in Fig. 5.5. The objective of the controller design is to minimize the error signal between $w(t)$ and the plant output $y(t)$. $K_{\rm fb}$ is a feedback gain to stabilize the plant, and Θ is a tunable parameter.

Note that if the system is invertible with properness as in [9, 10], then I can readily take $Q_\Theta = P^{-1}$ and obtain a perfect tracking. This is not the case, however, since I have obtained $P(s)$ in (5.8), which is strictly proper, and hence $P^{-1}(s)$ becomes improper. To overcome this difficulty, pre-filter $W(s)$ is introduced. Under a mild assumption on the plant, one can set the pre-filter to a diagonal form defined in (2.12).

A tunable feedforward controller is needed to be constructed. To generate $u_{\mathrm{ff}}(t)$, the dynamical system defined in (2.24) is considered. Then, the learning law (2.38) is used to tune $\Theta(t)$.

If $\Theta(t)$ converges to some constant value Θ_c which corresponds to F_c, G_c, H_c and $e(t) \to 0$ as $t \to \infty$, then I can obtain from (3.2) the following learning feedforward controller

$$\begin{aligned} u_{\mathrm{ff}_{\mathrm{Learning}}}(t) &= \left[I - G_c(sI - A_f)^{-1}B_f\right]^{-1} \\ &\quad \cdot \left\{H_c + F_c(sI - A_f)^{-1}B_f\right\} r(t) \\ &=: Q_{\Theta_c}(s) r(t). \end{aligned} \tag{5.12}$$

Clearly, the learning process for $Q_{\Theta_c}(s)$ depends on the reference signal $r(t)$. Thus, I employ a switching strategy in writing different characters since it means tracking to different references.

5.4. Simulation and Experimental Results

Using the on-line output $y(t)$ for learning may be the ultimate goal to demonstrate the practical usefulness of the proposed method. Toward this end, I first try a numerical simulation with nonlinear model (5.3). Since the nonlinear model includes the motor dynamics (5.3), I have to identify their parameters to perform the simulation under a realistic situation.

I use the following parameters for the length of the links and the equilibrium points in both simulation and experiment: $l_1 = 0.2$ [m], $l_2 = 0.2$ [m], $\theta_1^* = 30$ [deg], $\theta_2^* = 45$ [deg]. The working space is within 0.02 [m].

5.4.1 Identification of Motor Dynamics

As mentioned earlier, due to the double local feedback loops, the dynamics of each motor can be approximated as first order systems. I then identified the parameters for each motor via curve fitting method as

$$\theta_1 = \frac{0.9982}{0.2422s + 1} v_1,$$

$$\theta_2 = \frac{0.9956}{0.1458s + 1} v_1.$$

The responses are well approximated as shown in Figs. 5.6 and 5.7.

5.4.2 Simulation Results

I first conduct a numerical simulation based on the nonlinear model (5.3) and linearized model (5.8). I choose the feedback gain and diagonal pre-filter as

$$K_{\text{fb}} = \begin{bmatrix} -1 & 3 \\ -3 & -1 \end{bmatrix}, \quad W(s) = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}.$$

I also need to set stable A_f and B_f which makes (A_f, B_f) controllable based on the upper bound of the relative degree $\mu = 1$, (see (2.18) and (2.19)). They are chosen as

$$A_f = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}, \quad B_f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Note that the size of the matrix $\Theta(t)$ is 2 by 6. Thus, it includes 12 components. I consider the following two different reference trajectories to illustrate the idea clearly:

$$r_0(t) = \begin{bmatrix} \sin(0.4t) \\ \cos(0.4t) \end{bmatrix}, \quad r_8(t) = \begin{bmatrix} \sin(0.6t) \\ \sin(0.3t) \end{bmatrix}.$$

The reference trajectories represent the numerical numbers "0" and "8", respectively. I then tune $\Theta(t)$ by using the learning rule (2.38).

1) Nonlinear model case:

I restrict the working area of the hand position to within a small neighborhood around the equilibrium point to guarantee the convergence of the learning law. The time evolution of $\Theta_0(t)$ and $\Theta_8(t)$ are shown in Figs. 5.8 and 5.9, respectively. Each line in the plot corresponds to the time evolution of one component of the matrix over the time period. It can be seen that the convergence has been achieved only after a long time. The simulation results of the trajectories of the hand for "0" and "8" are shown in Fig. 5.10. It shows that the actual trajectories are close to the reference trajectories. As a result, one can verify that the algorithm really works for a nonlinear model with a restricted moving area. However, the result also shows that it takes a few days for convergence, even in the simulation. This implies that the experiment with on-line data via current algorithm is unrealistic. Thus, this direction of research will be considered in the future.

I now turn the attention toward the simulation with a linear model, as used in studies such as [9] and [10], to verify the proposed switching learning strategy in writing characters. The simulation result with the linear model is much faster than in the nonlinear case, as shown below.

2) Linear model case:

To solve the above problem of slow convergence, I use a linearized model of the plant, which can be obtained from (5.8) as follows:

$$P(s) = \begin{bmatrix} \frac{-0.04815}{0.2422s+1} & \frac{-0.05154}{0.1458s+1} \\ \frac{0.3657}{0.2422s+1} & \frac{-0.1923}{0.1458s+1} \end{bmatrix}. \tag{5.13}$$

Based on this model, I perform another numerical simulation. The time evolution of $\Theta_0(t)$ and $\Theta_8(t)$ is shown in Figs. 5.11 and 5.12, respectively. It is clear how fast the convergence is, compared to the nonlinear case. Thus, the effectiveness of using the linearized model is that the time elapsed for convergence is 0.67% of the time for the nonlinear case. The simulation results of the trajectories of the hand before and after learning are shown in Fig. 5.13. The resulting parameter matrices for each example from Figs. 5.11 and 5.12 are as follows:

a) for the first example ("0"):

$$F_0 = \begin{bmatrix} -0.2200 & 0.2806 \\ -0.5667 & -0.2726 \end{bmatrix}, \quad G_0 = \begin{bmatrix} 0.8815 & 1.2620 \\ 0.8662 & 2.8618 \end{bmatrix},$$

$$H_0 = \begin{bmatrix} -0.7721 & 1.3840 \\ -2.3745 & -0.9749 \end{bmatrix},$$

b) for the second example ("8"):

$$F_8 = \begin{bmatrix} -0.3847 & 0.4273 \\ -0.6492 & -0.2156 \end{bmatrix}, \quad G_8 = \begin{bmatrix} 2.9661 & 0.0107 \\ 0.0283 & 3.0981 \end{bmatrix},$$

$$H_8 = \begin{bmatrix} -1.6300 & 0.3725 \\ -2.8053 & -0.3475 \end{bmatrix},$$

where $e(t) \to 0$ for both. Thus, the resulting learning feedforward controller for each case using (5.20) is given as follows:

$$Q_{\Theta_0}(s) = \begin{bmatrix} \frac{-0.7721s^2-7.184s-16.47}{s^2+4.257s+2.456} & \frac{1.384s^2+6.162s+1.355}{s^2+4.257s+2.456} \\ \frac{-2.375s^2-18.14s-34.25}{s^2+4.257s+2.456} & \frac{-0.9749s^2-6.014s-7.973}{s^2+4.257s+2.456} \end{bmatrix},$$

$$Q_{\Theta_8}(s) = \begin{bmatrix} \frac{-1.63s^2-8.405s-6.354}{s^2+1.936s+0.9322} & \frac{0.3725s^2+2.25s+1.712}{s^2+1.936s+0.9322} \\ \frac{-2.805s^2-14.82s-12.47}{s^2+1.936s+0.9322} & \frac{-0.3475s^2-1.954s-1.606}{s^2+1.936s+0.9322} \end{bmatrix}.$$

To confirm that the above learning controllers can really let the manipulator write its corresponding characters, I perform an experiment with the real manipulator.

5.4.3 Experimental Results

The experimental setup of the two-link manipulator is shown in Fig. 5.1. The target is to let the manipulator write the numerical numbers 0, 2, 3,$\cdots$, 9. The first step is to obtain the learning controller for each character, as I did before for "0" and "8" with (5.13). I then switch the feedforward controller depending on the objective. For example, I use $Q_{\Theta_0}(s)$ if I want to write "0" and $Q_{\Theta_2}(s)$ to write "2" and so on. Note that I use the same K_{fb} and $W(s)$ in the simulation part to perform the experiment. Fig. 5.14 shows the picture of written "0" and "8" before and after learning. The resulting picture is close to the numerical simulation result in Fig. 5.13. The experimental result in Fig. 5.15 shows the reference trajectory versus the actual one for each number written by the manipulator. The results indicate that the manipulator succeeded in writing different characters based on the switching strategy.

5.5. Conclusion

The main objective of this work is to demonstrate the practical effectiveness of the MIMO-FEL scheme proposed in Chapter 2 by an experiment. I verified that controllers generated by the numerical simulation show good tracking performance in the experiment with a real two-link manipulator. The next goal is to use on-line data during the learning process. Under this configuration, I can be free from the identification of the motor dynamics and the linearization of nonlinear dynamics of the manipulator.

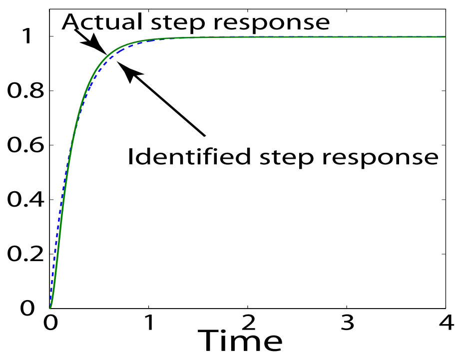


Figure 5.6. Motor 1 Identification

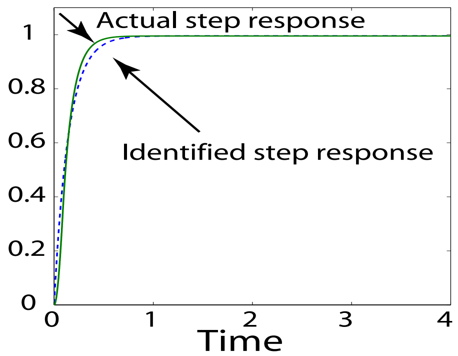


Figure 5.7. Motor 2 Identification

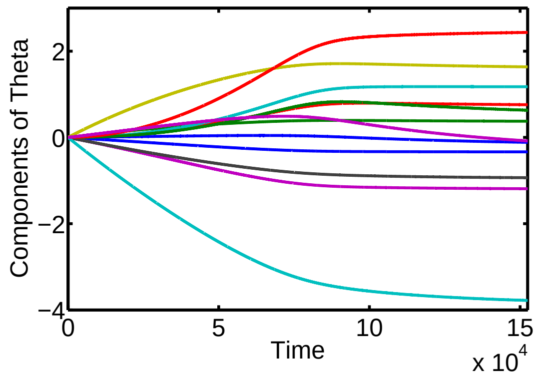


Figure 5.8. Time Evolution of Θ_0: Nonlinear Case

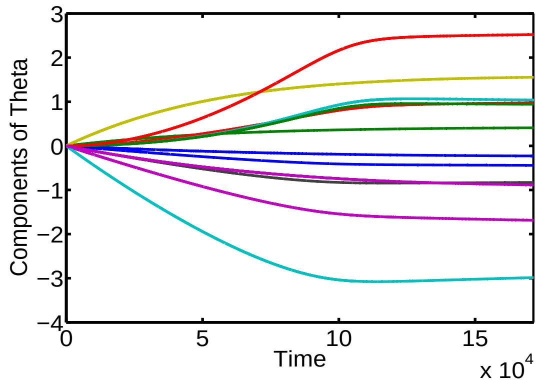


Figure 5.9. Time Evolution of Θ_8: Nonlinear Case

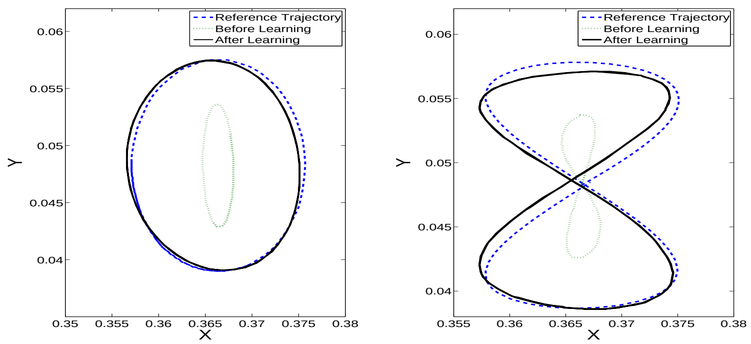


Figure 5.10. Nonlinear Simulation Results for 0 and 8

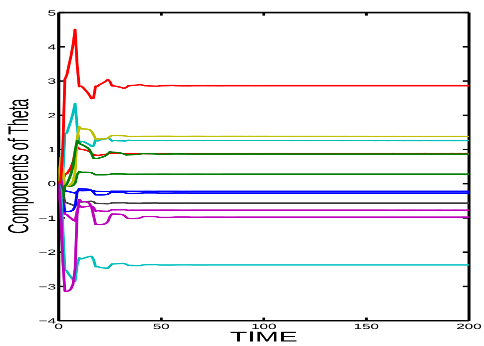


Figure 5.11. Time Evolution of Θ_0: Linear Case

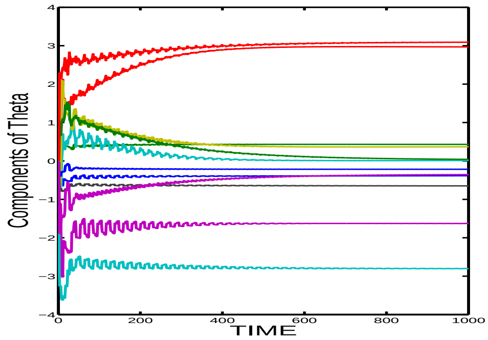


Figure 5.12. Time Evolution of Θ_8: Linear Case

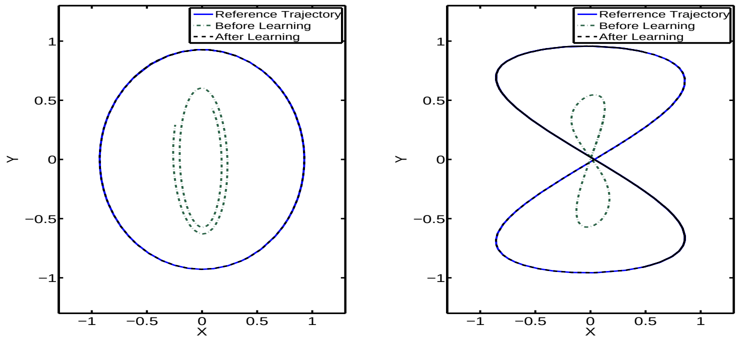


Figure 5.13. Linear Simulation Results for 0 and 8

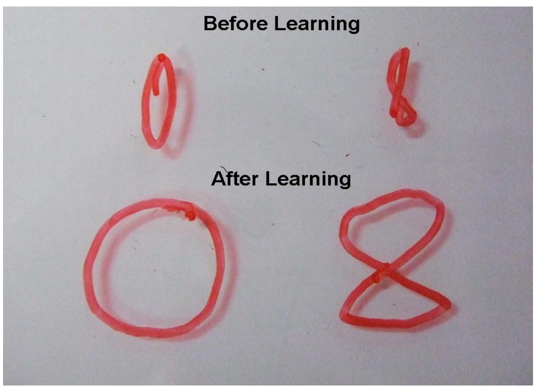
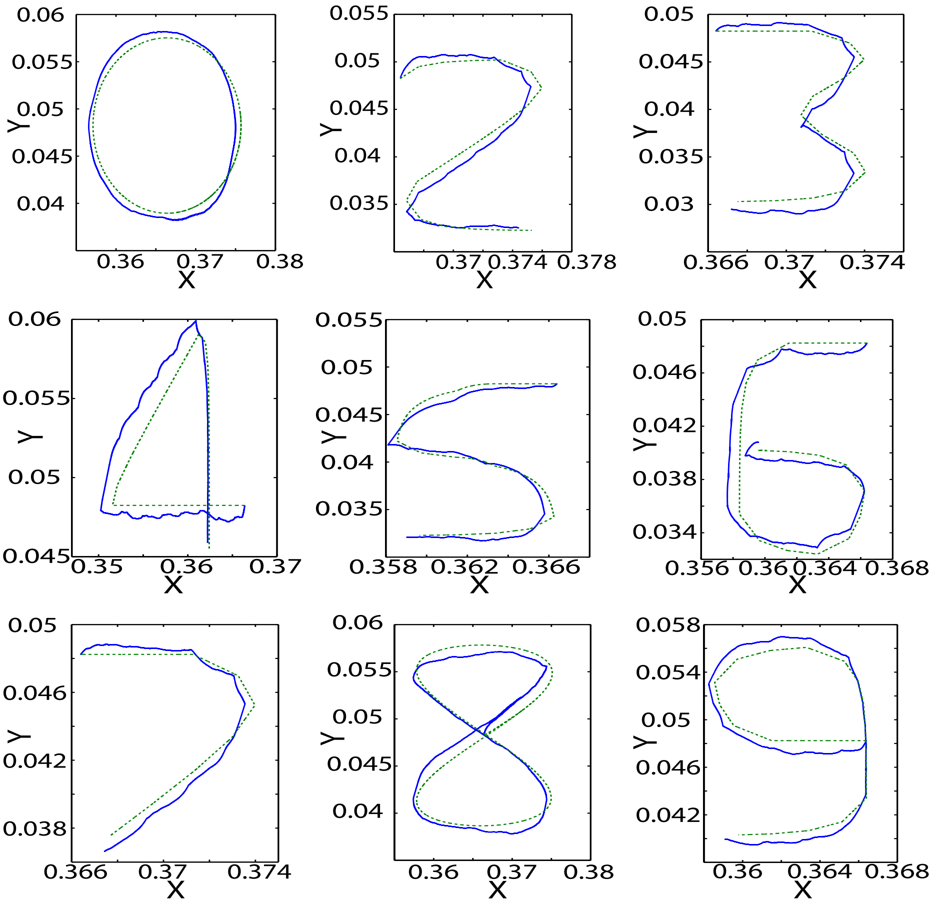


Figure 5.14. Characters 0 and 8 Written by the Manipulator



“- -” Reference Trajectory, “—” Actual Trajectory

Figure 5.15. Experimental Results of writing 0, 2, $\cdots$, 9

Chapter 6

Conclusion and Future Work

6.1. Conclusion

The ultimate goal of the thesis is to extend and apply FEL to MIMO systems. By using linear system parameterization as a function approximator of feedforward control, a learning law was derived to adjust the inverse of the plant. A theoretical treatment of how to generalize feedback error learning (FEL) to MIMO systems has been studied in the framework of adaptive control. The FEL scheme was generalized to MIMO systems which are not necessarily biproper, and hence not invertible with properness. The feedforward controller is not designed on the basis of the process model, rather trained on-line during control using feedback error signal as a learning signal.

Moreover, the exact inverse of the plant was derived theoretically using the inverse interactorization concept, and can be used as a guide to check the correctness of the learning control. Furthermore, the stability of the tuning rule has been proved with and without the positive realness condition.

In this thesis, I have also proposed a new closed-loop identification technique by means of MIMO-FEL. When the reference signal satisfied the PE condition, the method was shown to be effective in simulation.

Thus, the control performance can be improved by re-designing the controller with the obtained plant model. The convergence was slow, although this is often the case in adaptation or learning algorithms.

Further, I proved exponential convergence of the tracking error without considering the PE condition as presented in Chapter 2. The merit of this result is that in many practical applications good tracking performance is required, whereas it is not desirable or even impossible to satisfy the PE condition. Furthermore, parameter estimation using such benefits of FEL showed a better result than the conventional approach after the convergence without using the knowledge of the feedback controller, as required by most of the conventional methods in order to estimate the plant parameters.

Finally, I demonstrated the practical effectiveness of the proposed MIMO-FEL scheme by an experiment. I verified that controllers generated by numerical simulation showed good tracking performance in the experiment using the two-link manipulator.

6.2. Future Work

From the theoretical point of view, the results of this work can be extended to time delay systems which have a close relation to work done for the SISO case [16, 18] where the feedback loop has constant and unknown time delay. It is also desirable to extend the MIMO-FEL stability proof to include robust stability in the presence of plant uncertainty or unmodeled dynamics. The current experiment and simulation results in [51] showed the robustness of MIMO-FEL in the presence of motor and white noise, respectively. However, it requires further investigation to prove such results theoretically.

Since most of the adaptive control algorithms assumes the stability of the plant to be controlled, it would be interesting if the present work considers the case where the plant is unstable, but its inverse is stable. The simulation results show the capability of FEL to overcome such a difficult issue.

Moreover, one of interesting parts which needs to be improved in this thesis is the slow convergence of the closed-loop identification (scheme) in Chapter 3. Although I have introduced an alternative approach for parameter estimation using frequency response to overcome such a drawback, further investigations are needed to improve on the current work.

From the practical point of view, the next goal would be to perform the learning process on-line. Under this configuration, the identification of the motor dynamics and the linearization of nonlinear dynamics of the manipulator are not needed. To achieve this objective, the current work must be extended to cover the nonlinear model by finding a proper mathematical representation of a nonlinear filter instead of using linear one.

Appendix

Strictly Positive Real (SPR) Definition for Biproper MIMO System [48]

The $m \times m$ biproper transfer function matrix $G(s) = D + C(sI - A)^{-1}B$ with $D > 0$, is called SPR if:

1. All elements of $G(s)$ are analytic in $Re(s) \geq 0$ (*i.e.*, they do not have a pole in $Re[s] \geq 0$.)
2. $G(s)$ is real for real s.
3. $G(s) + G^*(s) > 0$ for $Re[s] \geq 0$.

Strictly Positive Real (SPR) Definition for Strictly Proper MIMO System [48]

The $m \times m$ strictly proper transfer function matrix $G(s) = C(sI - A)^{-1}B$, is called SPR if:

1. All elements of $G(s)$ are analytic in $Re(s) \geq 0$.
2. $G(s)$ is real for real s.
3. $G(s) + G^*(s) \geq 0$ for $Re[s] \geq 0$, $G(s) + G^*(s) > 0$ for finite s, and

$$\lim_{\omega \to \infty} \left[\omega^2 \left(G(j\omega) + G^*(j\omega) \right) \right] > 0.$$

KYP Lemma (Kalman-Yakubovich-Popov) [48]

Let $G(s) = D + C(sI - A)^{-1}B$,
be $m \times m$ transfer matrix, where (A, B) is controllable and (A, C) is observable. Then, $G(s)$ is strictly positive real if and only if there exist matrices $P = P^T$, T, and Z, and a positive constant ϵ such that

$$\begin{aligned}
&PA + A^T P = -T^T T - \epsilon P, \\
&PB = C^T - T^T Z, \\
&Z^T Z = D + D^T.
\end{aligned}$$

Persistent Excitation (PE) Condition [34]

A vector $\zeta(t)$ is PE if there exist λ_1, λ_2,$\delta > 0$ such that
$\lambda_1 I \geq \int_{t_0}^{t_0+\delta} \zeta(t)\zeta^T(t)dt \geq \lambda_2 I$, for all $t_0 \geq 0$.

PE and Exponential Stability Theorem [34]

Let $w(t)$ be piecewise continuous.
If w is PE, then $\dot{\phi}(t) = -gw(t)w^T(t)\phi(t)$ for $g > 0$ is globally exponentially stable.

Barbalat Lemma [4]

If g is a real function of a real variable t, defined and uniformly continuous for $t \geq 0$, and if the limit of the integral

$$\int_0^t g(s)ds$$

as t tends to infinity exists and is a finite number, then

$$\lim_{t \to \infty} g(t) = 0.$$

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